Export and Hedging Decisions under Correlated Revenue and Exchange Rate Risk

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JEL classification: D21; D24; D81; F31

Keywords: Background risk; Futures hedging; Production; Prudence

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Abstract

This paper examines the behavior of a competitive exporting firm that faces joint revenue and exchange rate risk. The firm can trade unbiased currency futures contracts for hedging purposes. We show that the correlation between the revenue shock and the random spot exchange rate plays a pivotal role in determining the firm's optimal export and hedging decisions. If the correlation is either zero or negative, the firm optimally produces less than the benchmark level in the absence of the revenue shock, and under-hedges its exchange rate risk exposure. However, if the correlation is positive, we derive sufficient conditions under which the firm optimally produces more, not less, than the benchmark level. The firm as such uses operational and financial hedging as complements to better cope with the multiple sources of uncertainty.

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1. Introduction

The literature on the behavior of a competitive exporting firm under exchange rate uncertainty is abundant. One important strand of the literature studies the firm's optimal export and hedging decisions when a currency futures market exists (see, e.g., Benninga et al., 1985; Broll and Zilcha, 1992; Katz and Paroush, 1979; Kawai and Zilcha, 1986; Viaene and Zilcha, 1998; Wong, 2003a; to name just a few). Two notable results emanate. First, the separation theorem states that the firm's optimal production decision depends neither on the risk attitude of the firm nor on the incidence of the underlying exchange rate uncertainty. Second, the full-hedging theorem states that the firm should completely eliminate its exchange rate risk exposure by adopting a full-hedge should the currency futures market be unbiased.[1]

The purpose of this paper is to reexamine the separation and full-hedging theorems when the competitive exporting firm is confronted with not only the exchange rate uncertainty but also a revenue shock as in Adam-Müller (1997). Revenue shocks may come from various sources: uncertain output prices, political uncertainty in the foreign country, credit risk of the importing firm, and many others. Unlike Adam-Müller (1997), we allow the revenue shock faced by the firm to possibly be correlated with the exchange rate uncertainty. For example, the prevalence of incomplete exchange rate pass-through is likely to suggest a negative correlation between the random spot exchange rate and the prevailing output price (see Wong, 2003b). Using annual political and exchange rate risk ratings from the International Country Risk Guide of the Political Risk Services, Kim and Song (2006) document that these two measures are statistically significantly positive. While the exchange rate risk is hedgeable by trading currency futures contracts, the revenue shock is neither hedgeable nor insurable.

We show that the separation theorem is invalidated when the revenue shock prevails. The correlation between the random spot exchange rate and the revenue shock plays a pivotal role in determining the firm's optimal output level as compared to the benchmark level when the revenue shock is absent. Specifically, if the correlation is either zero or negative, the firm optimally produces less and shirks its export to the foreign country so as to limit its exposure to the residual risk that is unhedgeable by trading the currency futures contracts. However, if the correlation is positive, we derive sufficient conditions under which the firm optimally produces more, not less, than the benchmark level. Hence, the firm uses operational and financial hedging as complements to better cope with the multiple sources of uncertainty.

The full-hedging theorem holds in the presence of the revenue shock only under restrictive conditions: (i) the firm has a quadratic utility function, and (ii) the revenue shock is

[1] The full-hedging theorem is analogous to a well-known result in the insurance literature that a risk-averse individual fully insures at an actuarially fair price (see Mossin, 1968).

independent of the random spot exchange rate. When the two random variables are negatively (positively) correlated, the firm is induced to opt for a long (short) futures position, thereby rendering the optimality of an under-hedge (over-hedge). If the firm's preferences satisfy prudence in the sense of Kimball (1990, 1993), a precautionary motive arises that always induces the firm to opt for a long futures position. The prudent firm's optimal futures position is thus an under-hedge if the revenue shock and the random spot exchange rate are either independent or negatively correlated. Should the two random variables be positively correlated, the precautionary motive counteracts the correlation motive, thereby making the prudent firm's optimal futures position indeterminate.

The rest of this paper is organized as follows. Section 2 delineates the model of a competitive exporting firm under joint revenue and exchange rate risk. The firm can trade unbiased currency futures contracts for hedging purposes. Section 3 considers a benchmark case in which the revenue risk is absent. Section 4 examines the firm's optimal export and hedging decisions. The final section concludes.

2. The model

Consider a competitive exporting firm that operates for one period with two dates, 0 and 1. The firm possesses a von Neumann-Morgenstern utility function, $U(\Pi)$, defined over its home currency profit at date 1, Π. The firm is risk averse so that $U'(\Pi) > 0$ and $U''(\Pi) < 0$ for all $\Pi > 0$.

To begin, the firm produces a single commodity according to a deterministic cost function, $C(Q)$, in the home country, where $Q \geq 0$ is the output level and $C(Q)$ is compounded to date 1. The firm's production technology exhibits decreasing returns to scale so that $C(Q)$ satisfies that $C(0) = C'(0) = 0$, and that $C'(Q) > 0$ and $C''(Q) > 0$ for all $Q > 0$. The firm sells its entire output in a foreign country at a per-unit price, P, at date 1, where $P > 0$ is exogenously fixed and denominated in the foreign currency. As in Adam-Müller

(1997), the firm's foreign currency revenue, $\tilde{\theta}PQ$, is subject to a shock, $\tilde{\theta}$, which is a positive random variable with unit mean.[2] The spot exchange rate, $\tilde{S}$, which is expressed in units of the home currency per unit of the foreign currency at date 1, is also a positive random variable. Unlike Adam-Müller (1997), the two random variables, $\tilde{\theta}$ and $\tilde{S}$, are not necessarily independent.

While the revenue shock, $\tilde{\theta}$, is neither hedgeable nor insurable, the firm can hedge its exchange rate risk exposure to $\tilde{S}$ by trading infinitely divisible currency futures contracts at date 0, each of which calls for delivery of S^f units of the home currency per unit of the foreign currency. The firm's random profit at date 1, denominated in the home currency, is given by

$$\tilde{\Pi} = \tilde{\theta}\tilde{S}PQ + (S^f - \tilde{S})H - C(Q), \tag{1}$$

where H is the number of the currency futures contracts sold (purchased if negative) by the firm at date 0. The futures position, H, is said to be an under-hedge, a full-hedge, or an over-hedge, depending on whether H is smaller than, equal to, or greater than the expected foreign currency revenue, PQ, respectively.

The firm's ex-ante decision problem is to choose an output level, $Q \geq 0$, and a futures position, H, at date 0 so as to maximize the expected utility of its home currency profit at date 1:

$$\max_{Q\geq 0, H} \ \mathrm{E}[U(\tilde{\Pi})], \tag{2}$$

where $\mathrm{E}(\cdot)$ is the expectation operator with respect to the joint cumulative distribution function of $\tilde{\theta}$ and $\tilde{S}$, and $\tilde{\Pi}$ is given by Eq. (1). The first-order conditions for program (2) are given by

$$\mathrm{E}\{U'(\tilde{\Pi}^*)[\tilde{\theta}\tilde{S}P - C'(Q^*)]\} = 0, \tag{3}$$

[2] Throughout the paper, random variables have a tilde (~) while their realizations do not.

and

$$\mathrm{E}[U'(\tilde{\Pi}^*)(S^f - \tilde{S})] = 0, \tag{4}$$

where an asterisk (*) signifies an optimal level.[3]

To focus on the firm's pure hedging motive, we assume hereafter that the currency futures contracts are unbiased in that $S^f = \mathrm{E}(\tilde{S})$. We can then write Eqs. (3) and (4) as[4]

$$\mathrm{E}(\tilde{S})P + \mathrm{Cov}(\tilde{\theta}, \tilde{S})P - C'(Q^*) = -\frac{\mathrm{Cov}[U'(\tilde{\Pi}^*), \tilde{\theta}\tilde{S}]P}{\mathrm{E}[U'(\tilde{\Pi}^*)]}, \tag{5}$$

and

$$\mathrm{Cov}[U'(\tilde{\Pi}^*), \tilde{S}] = 0, \tag{6}$$

where $\mathrm{Cov}(\cdot, \cdot)$ is the covariance operator with respect to the joint cumulative distribution function of $\tilde{\theta}$ and $\tilde{S}$. Eq. (5) takes into account the fact that $\mathrm{E}(\tilde{\theta}) = 1$.

3. A benchmark case without revenue risk

As a benchmark, we consider in this section the case that there is no revenue shock, i.e., $\tilde{\theta} \equiv 1$. In this benchmark case, Eqs. (5) and (6) become

$$\mathrm{E}(\tilde{S})P - C'(Q^\circ) = -\frac{\mathrm{Cov}[U'(\tilde{\Pi}^\circ), \tilde{S}]P}{\mathrm{E}[U'(\tilde{\Pi}^\circ)]}, \tag{7}$$

and

$$\mathrm{Cov}[U'(\tilde{\Pi}^\circ), \tilde{S}] = 0, \tag{8}$$

[3]The second-order conditions for program (2) are satisfied given risk aversion and the strict convexity of the cost function.

[4]For any two random variables, $\tilde{X}$ and $\tilde{Y}$, we have $\mathrm{Cov}(\tilde{X}, \tilde{Y}) = \mathrm{E}(\tilde{X}\tilde{Y}) - \mathrm{E}(\tilde{X})\mathrm{E}(\tilde{Y})$.

respectively, where a nought ($^\circ$) signifies an optimal level, and $\tilde{\Pi}^\circ$ is given by Eq. (1) with $\tilde{\theta} \equiv 1$. Solving Eqs. (7) and (8) simultaneously yields our first proposition.

Proposition 1. *Given that the currency futures contracts are unbiased, and that the revenue shock is absent, the competitive exporting firm's optimal output level, Q°, is the unique solution to*

$$\mathrm{E}(\tilde{S})P = C'(Q^\circ), \tag{9}$$

and its optimal futures position, H°, is a full-hedge, i.e., $H^\circ = PQ^\circ$.

Proof. Substituting Eq. (8) into Eq. (7) yields Eq. (9). Suppose that $H^\circ = PQ^\circ$. The firm's home currency profit at date 1 becomes $\tilde{\Pi}^\circ = \mathrm{E}(\tilde{S})PQ^\circ - C(Q^\circ)$, which is non-stochastic. In this case, Eq. (8) is satisfied, thereby implying that $H^\circ = PQ^\circ$ is indeed the firm's optimal futures position. □

To see the intuition for Proposition 1, we recast Eq. (1) with $\tilde{\theta} \equiv 1$ and $S^f = \mathrm{E}(\tilde{S})$ as

$$\tilde{\Pi} = \mathrm{E}(\tilde{S})PQ - C(Q) + [\tilde{S} - \mathrm{E}(\tilde{S})](PQ - H). \tag{10}$$

Inspection of Eq. (10) reveals that the firm could have completely eliminated its exchange rate risk exposure had it chosen a full-hedge, i.e., $H = PQ$, within its own discretion. Alternatively put, the degree of exchange rate risk exposure to be assumed by the firm should be totally unrelated to its production decision. The firm as such chooses the optimal output level, Q°, that maximizes $\mathrm{E}(\tilde{S})PQ - C(Q)$, which gives rise to Eq. (9). Since the currency futures contracts are unbiased, the firm finds it optimal to completely eliminate its exchange rate risk exposure by adopting a full-hedge, i.e., $H^\circ = PQ^\circ$. These results are simply the celebrated separation and full-hedging theorems emanated from the literature on international firms under exchange rate uncertainty.

4. Optimal export and hedging decisions

In this section, we resume the original case in which the revenue risk prevails. Note that

$$\begin{aligned}\mathrm{Cov}[U'(\tilde{\Pi}^*),\tilde{\theta}\tilde{S}] &= \mathrm{Cov}[U'(\tilde{\Pi}^*),\tilde{S}] + \mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S},\tilde{\theta}] - \mathrm{E}[U'(\tilde{\Pi}^*)]\mathrm{Cov}(\tilde{\theta},\tilde{S}) \\ &= \mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S},\tilde{\theta}] - \mathrm{E}[U'(\tilde{\Pi}^*)]\mathrm{Cov}(\tilde{\theta},\tilde{S}), \end{aligned} \tag{11}$$

where the second equality follows from Eq. (6). Substituting Eq. (11) into Eq. (5) yields

$$\mathrm{E}(\tilde{S})P - C'(Q^*) = -\frac{\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S},\tilde{\theta}]P}{\mathrm{E}[U'(\tilde{\Pi}^*)]}. \tag{12}$$

It follows from Eqs. (9) and (12) and the strict convexity of the cost function that $Q^* > (<)\ Q^\circ$ if $\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S},\tilde{\theta}] > (<)\ 0$.

Since the revenue shock is neither hedgeable nor insurable, the firm's home currency profit at date 1 must be stochastic. It then follows from Eq. (1) that

$$\begin{aligned}\mathrm{Cov}[U'(\tilde{\Pi}^*),\tilde{\Pi}^*] &= \mathrm{Cov}[U'(\tilde{\Pi}^*),\tilde{\theta}\tilde{S}]PQ^* - \mathrm{Cov}[U'(\tilde{\Pi}^*),\tilde{S}]H^* \\ &= \mathrm{Cov}[U'(\tilde{\Pi}^*),\tilde{\theta}\tilde{S}]PQ^* < 0, \end{aligned} \tag{13}$$

where the second equality follows from Eq. (6), and the the inequality follows from $U''(\Pi) < 0$. If $\mathrm{Cov}(\tilde{\theta},\tilde{S}) \leq 0$, Eqs. (11) and (13) imply that $\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S},\tilde{\theta}] < 0$, thereby invoking the following proposition.

Proposition 2. *Given that the currency futures contracts are unbiased, the competitive exporting firm's optimal output level, Q^*, is less than Q° if the revenue shock, $\tilde{\theta}$, and the random spot exchange rate, $\tilde{S}$, are either uncorrelated or negatively correlated.*

To see the intuition for Proposition 2, we use Eq. (1) with $S^f = \mathrm{E}(\tilde{S})$ to write

$$\tilde{\Pi}^* = \mathrm{E}(\tilde{S})PQ^* - C(Q^*) + (\tilde{\theta}-1)\tilde{S}PQ^* + [\tilde{S} - \mathrm{E}(\tilde{S})](PQ^* - H^*). \tag{14}$$

The prevalence of the revenue shock induces the firm to cut down its output so as to limit the risk exposure that comes from the third term on the right-hand side of Eq. (14). Taking expectations on both sides of Eq. (14) yields

$$\mathrm{E}(\tilde{\Pi}^*) = \mathrm{E}(\tilde{S})PQ^* - C(Q^*) + \mathrm{Cov}(\tilde{\theta}, \tilde{S})PQ^*. \tag{15}$$

If $\tilde{\theta}$ and $\tilde{S}$ are negatively correlated (uncorrelated), the last term on the right-hand side of Eq. (15) is decreasing in (invariant to) output, which reinforces (has no effect on) the firm's risk reduction incentive, thereby rendering $Q^* < Q^\circ$.

When $\mathrm{Cov}(\tilde{\theta}, \tilde{S}) > 0$, the last term on the right-hand side of Eq. (15) is increasing in the firm's output level. This counteracts the firm's risk reduction incentive. Hence, if the positive correlation between $\tilde{\theta}$ and $\tilde{S}$ is sufficiently large, it is possible that the firm may produce more, not less, than the optimal output level in the benchmark case of no revenue shock. To show the possibility that $Q^* > Q^\circ$, we follow Briys et al. (1993) and Wong (2003b) to assume that $\tilde{\theta}$ and $\tilde{S}$ are related in the following manner:

$$\tilde{\theta} = 1 + \beta[\tilde{S} - \mathrm{E}(\tilde{S})] + \tilde{\varepsilon}, \tag{16}$$

where β is a constant, and $\tilde{\varepsilon}$ is a zero-mean random variable independent of $\tilde{S}$. According to Eq. (16), $\tilde{\theta}$ and $\tilde{S}$ are negatively or positively correlated depending on whether β is negative or positive, respectively. We derive sufficient conditions under which $Q^* > Q^\circ$ in the following proposition.

Proposition 3. *Suppose that the competitive exporting firm has access to the unbiased currency futures contracts for hedging purposes, and that the firm possesses a quadratic utility function, $U(\Pi) = a\Pi - \Pi^2/2$, where a is a positive constant such that $a - \Pi > 0$ for the relevant range of Π. If the two random variables, $\tilde{\theta}$ and $\tilde{S}$, are characterized by Eq. (16) with $\beta > 0$, and if*

$$\mathrm{Cov}(\tilde{\theta}, \tilde{S}) \geq \left[\frac{PQ^\circ}{a - \mathrm{E}(\tilde{S})PQ^\circ + C(Q^\circ)}\right]\mathrm{E}[(\tilde{\theta} - 1)^2\tilde{S}^2], \tag{17}$$

where Q° is the optimal output level in the benchmark case of no revenue shock, the firm's optimal output level, Q^*, is greater than Q°.

Proof. See Appendix A. □

Since $\mathrm{E}[(\tilde{\theta}-1)^2\tilde{S}^2] > 0$ and $a > \mathrm{E}(\tilde{S})PQ^\circ - C(Q^\circ)$, where Q° solves Eq. (9), the right-hand side of condition (17) is strictly positive. Condition (17) states that the firm optimally produces more, i.e., $Q^* > Q^\circ$, when the revenue shock prevails should the positive correlation between $\tilde{\theta}$ and $\tilde{S}$ be large enough. We can rewrite condition (17) as

$$a \geq \left\{\mathrm{E}(\tilde{S}) + \frac{\mathrm{E}[(\tilde{\theta}-1)\tilde{S}^2]}{\mathrm{Cov}(\tilde{\theta}, \tilde{S})}\right\}PQ^\circ - C(Q^\circ). \tag{18}$$

Hence, if the firm is not too risk averse, i.e., a is sufficiently large that condition (18) holds, then $Q^* > Q^\circ$ given that $\tilde{\theta}$ and $\tilde{S}$ are positively correlated.

We now examine the firm's optimal futures position, H^*. For tractability, we assume that $\tilde{\theta}$ and $\tilde{S}$ are characterized by Eq. (16). Consider first that the firm possesses a quadratic utility function, which, without any loss of generality, is specified as $U(\Pi) = a\Pi - \Pi^2/2$, where a is a positive constant such that $a - \Pi > 0$ for the relevant range of Π. Using $U'(\Pi) = a - \Pi$ and Eq. (16), we can write Eq. (6) as

$$\mathrm{Cov}\left\{\{1 + \beta[\tilde{S} - \mathrm{E}(\tilde{S})] + \tilde{\varepsilon}\}\tilde{S}PQ^* - \tilde{S}H^*, \tilde{S}\right\} = 0. \tag{19}$$

Since $\tilde{\varepsilon}$ is independent of $\tilde{S}$, we can rewrite Eq. (19) as[5]

$$H^* - PQ^* = \frac{\beta PQ^*\mathrm{E}\{[\tilde{S} - \mathrm{E}(\tilde{S})]^2\tilde{S}\}}{\mathrm{Var}(\tilde{S})}. \tag{20}$$

where $\mathrm{Var}(\tilde{S}) > 0$ is the variance of $\tilde{S}$. Since $\mathrm{E}\{[\tilde{S} - \mathrm{E}(\tilde{S})]^2\tilde{S}\} > 0$, Eq. (20) implies that $H^* < (>)\ PQ^*$ if $\beta < (>)\ 0$. Hence, we establish the following proposition.

[5]Note that $\mathrm{Cov}(\tilde{\varepsilon}\tilde{S}, \tilde{S}) = \mathrm{E}(\tilde{\varepsilon}\tilde{S}^2) - \mathrm{E}(\tilde{\varepsilon}\tilde{S})\mathrm{E}(\tilde{S}) = 0$.

Proposition 4. *Suppose that the competitive exporting firm with a quadratic utility function has access to the unbiased currency futures contracts for hedging purposes, and that the two random variables, $\tilde{\theta}$ and $\tilde{S}$, are characterized by Eq. (16). The firm optimally opts for an under-hedge (over-hedge), i.e., $H^* < (>)\ PQ^*$, if $\tilde{\theta}$ and $\tilde{S}$ are negatively (positively) correlated, i.e., $\beta < (>)\ 0$. A full-hedge, i.e., $H^* = PQ^*$, is optimal if $\tilde{\theta}$ and $\tilde{S}$ are independent, i.e., $\beta = 0$.*

The intuition for Proposition 4 is as follows. Given that covariances can be interpreted as marginal variances, Eq. (6) implies that the optimal futures position, H^*, is the one that minimizes the variance of the firm's marginal utility, $U'(\tilde{\Pi}^*)$. Given a quadratic utility function, this is tantamount to minimizing the variability of the firm's home currency profit at date 1, which is given by Eq. (14). If $\beta = 0$, the risk exposure that comes from the third term on the right-hand side of Eq. (14) is not hedgeable by trading the currency futures contracts. To minimize the risk exposure due to the last term on the right-hand side of Eq. (14), the firm optimally opts for a full-hedge, i.e., $H^* = PQ^*$. We refer to this as the full-hedging motive. If $\beta < (>)\ 0$, then $\mathrm{Cov}[(\tilde{\theta} - 1)\tilde{S}, \tilde{S}] = \beta \mathrm{E}\{[\tilde{S} - \mathrm{E}(\tilde{S})]^2 \tilde{S}\} < (>)\ 0$. The firm as such is induced to opt for a long (short) futures position so as to reduce the risk exposure arising from the third term on the right-hand side of Eq. (14). We refer to this as the correlation motive. Combining the full-hedging and correlation motives gives rise to the optimality of an under-hedge (over-hedge), i.e., $H^* < (>)\ PQ^*$, if $\beta < (>)\ 0$.

As convincingly argued by Kimball (1990, 1993), prudence, i.e., $U'''(\Pi) > 0$, is a reasonable behavioral assumption for decision making under multiple sources of uncertainty. Prudence measures the propensity to prepare and forearm oneself under uncertainty, vis-à-vis risk aversion that is how much one dislikes uncertainty and would turn away from it if one could. As is shown by Drèze and Modigliani (1972), Kimball (1990), and Leland (1968), prudence is both necessary and sufficient to induce precautionary saving. Furthermore, prudence is implied by decreasing absolute risk aversion, which is instrumental in yielding many intuitively appealing comparative statics under uncertainty (Gollier, 2001).

Hence, it is of great interest to examine the optimal futures position, H^*, of the prudent firm, which is done in the following proposition.

Proposition 5. *Suppose that the competitive exporting firm is prudent and has access to the unbiased currency futures contracts for hedging purposes, and that the two random variables, $\tilde{\theta}$ and $\tilde{S}$, are characterized by Eq. (16). The firm optimally opts for an under-hedge, i.e., $H^* < PQ^*$, if $\tilde{\theta}$ and $\tilde{S}$ are either independent or negatively correlated, i.e., $\beta \leq 0$.*

Proof. See Appendix B. □

The intuition for Proposition 5 is as follows. Since the revenue shock has unit mean, the random variable, $\tilde{\theta} - 1$, that appears in the third term on the right-hand side of Eq. (14) can be interpreted as a zero-mean background risk. The firm, being prudent, has a precautionary motive to shift its profit from states with small background risk to states with large background risk so as to mitigate the loss of utility (see Eeckhoudt and Schlesinger, 2006). As is evident from the third term on the right-hand side of Eq. (14), the magnitude of the background risk increases with an increase in the realized value of $\tilde{S}$. The precautionary motive as such calls for a long futures position that shifts the firm's profit from states with small background risk to states with large background risk. If $\beta <$ ($>$) 0, the correlation motive reinforces (counteracts) the precautionary motive. Combining these two motives with the full-hedging motive, the prudent firm finds it optimal to opt for an under-hedge, i.e., $H^* < PQ^*$, if $\beta \leq 0$, and the optimal futures position becomes ambiguous if $\beta > 0$.

To illustrate the results of Proposition 5, we consider the following example. Suppose that the random spot exchange rate, $\tilde{S}$, can take on two possible values, $\underline{S}$ and $\overline{S}$, with $0 < \underline{S} < \overline{S}$. Let p be the probability that $\tilde{S} = \underline{S}$, where $0 < p < 1$. The expected spot exchange rate is therefore given by $\mathrm{E}(\tilde{S}) = p\underline{S} + (1-p)\overline{S}$. The revenue shock, $\tilde{\theta}$, is related to $\tilde{S}$ according to Eq. (16), where $\tilde{\varepsilon}$ is assumed to be a standard normal random variable.

The firm's preferences exhibit constant absolute risk aversion, $U(\Pi) = -e^{-\alpha\Pi}$, where $\alpha > 0$ is the constant coefficient of absolute risk aversion. For simplicity, we fix the firm's output level at Q^*.

The first-order condition for this two-state example is given by

$$p\alpha \mathrm{E}\{e^{-\alpha\{\{1+\beta[\underline{S}-\mathrm{E}(\tilde{S})]+\tilde{\varepsilon}\}\underline{S}PQ^*+[\mathrm{E}(\tilde{S})-\underline{S}]H^*-C(Q^*)\}}\}[\mathrm{E}(\tilde{S})-\underline{S}]$$

$$+(1-p)\alpha \mathrm{E}\{e^{-\alpha\{\{1+\beta[\overline{S}-\mathrm{E}(\tilde{S})]+\tilde{\varepsilon}\}\overline{S}PQ^*+[\mathrm{E}(\tilde{S})-\overline{S}]H^*-C(Q^*)\}}\}[\mathrm{E}(\tilde{S})-\overline{S}] = 0, \qquad (21)$$

where H^* is the firm's optimal futures position. Since $\mathrm{E}(\tilde{S}) = p\underline{S} + (1-p)\overline{S}$, Eq. (21) reduces to

$$e^{-\alpha\{[1-\beta(1-p)(\overline{S}-\underline{S})]\underline{S}PQ^*+(1-p)(\overline{S}-\underline{S})H^*-C(Q^*)-\alpha(\underline{S}PQ^*)^2/2\}}$$

$$= e^{-\alpha\{[1+\beta p(\overline{S}-\underline{S})]\overline{S}PQ^*-p(\overline{S}-\underline{S})H^*-C(Q^*)-\alpha(\overline{S}PQ^*)^2/2\}}, \qquad (22)$$

where we have used the fact that $\tilde{\varepsilon}$ is a standard normal random variable. Solving Eq. (22) yields

$$H^* = \left\{1 + \beta[p\overline{S} + (1-p)\underline{S}] - \frac{\alpha}{2}PQ^*(\underline{S} + \overline{S})\right\}PQ^*. \qquad (23)$$

It is evident from Eq. (23) that $H^* < PQ^*$ if $\beta \leq 0$, which is consistent with the results of Proposition 5. Even when $\beta > 0$, it follows from continuity and Eq. (23) that $H^* < PQ^*$ as long as β is not too positive.[6]

5. Conclusion

In this paper, we examine the behavior of a competitive exporting firm under joint revenue and exchange rate risk. We show that the separation theorem does not hold when

[6]Since a change in β affects the optimal output level, Q^*, we need to simultaneously determine Q^* and H^* in order to solve the threshold value of β above which $H^* > PQ^*$.

the revenue shock prevails. The full-hedging theorem holds only under restrictive conditions: (i) the firm has a quadratic utility function, and (ii) the revenue shock is independent of the random spot exchange rate. The correlation between the revenue shock and the random spot exchange rate plays a pivotal role in determining the firm's optimal export and hedging decisions. If the correlation is either zero or negative, the firm optimally produces less than the benchmark level in the absence of the revenue shock, and under-hedges its exchange rate risk exposure. However, if the correlation is positive, we derive sufficient conditions under which the firm optimally produces more, not less, than the benchmark level. The firm as such uses operational and financial hedging as complements to better cope with the multiple sources of uncertainty.

Appendix A. Proof of Proposition 3

Consider a hypothetical case in which there are forward contracts for the revenue shock, $\tilde{\theta}$. The forward price is set equal to $\mathrm{E}(\tilde{\theta}) = 1$. In this hypothetical case, the firm's random home currency profit at date 1 is given by

$$\tilde{\Pi} = \tilde{\theta}\tilde{S}PQ + [\mathrm{E}(\tilde{S}) - \tilde{S}]H + (1 - \tilde{\theta})\tilde{S}Z - C(Q), \tag{A.1}$$

where Z is the number of the forward contracts for $\tilde{\theta}$ sold (purchased if negative) by the firm. The firm's ex-ante decision problem is to choose an output level, $Q \geq 0$, a futures position, H, and a forward position, Z, at date 0 so as to maximize the expected utility of its home currency profit at date 1:

$$\max_{Q \geq 0, H, Z} \ \mathrm{E}[U(\tilde{\Pi})], \tag{A.2}$$

where $\tilde{\Pi}$ is given by Eq. (A.1). The first-order conditions for program (A.2) are given by

$$\mathrm{E}(\tilde{S})P + \mathrm{Cov}(\tilde{\theta}, \tilde{S})P - C'(Q^{\diamond}) = -\frac{\mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{\theta}\tilde{S}]P}{\mathrm{E}[U'(\tilde{\Pi}^{\diamond})]}, \tag{A.3}$$

$$\mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{S}] = 0, \tag{A.4}$$

and

$$\mathrm{Cov}[U'(\tilde{\Pi}^{\diamond})\tilde{S}, \tilde{\theta}] = 0, \tag{A.5}$$

where a diamond ($^{\diamond}$) signifies an optimal level. Note that

$$\begin{aligned}\mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{\theta}\tilde{S}] &= \mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{S}] + \mathrm{Cov}[U'(\tilde{\Pi}^{\diamond})\tilde{S}, \tilde{\theta}] - \mathrm{E}[U'(\tilde{\Pi}^{\diamond})]\mathrm{Cov}(\tilde{\theta}, \tilde{S}) \\ &= -\mathrm{E}[U'(\tilde{\Pi}^{\diamond})]\mathrm{Cov}(\tilde{\theta}, \tilde{S}). \qquad (A.6)\end{aligned}$$

where the second equality follows from Eqs. (A.4) and (A.5). Substituting Eq. (A.6) into Eq. (A.3) yields $\mathrm{E}(\tilde{S})P = C'(Q^{\diamond})$. It then follows from Eq. (9) that $Q^{\diamond} = Q^{\circ}$.

Since the firm's home currency profit at date 1 is stochastic, it follows from Eq. (A.1) that

$$\begin{aligned}\mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{\Pi}^{\diamond}] &= \mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{\theta}\tilde{S}](PQ^{\circ} - Z^{\diamond}) + \mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{S}](Z^{\diamond} - H^{\diamond}) \\ &= \mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{\theta}\tilde{S}](PQ^{\circ} - Z^{\diamond}) < 0, \qquad (A.7)\end{aligned}$$

where the second equality follows from Eq. (A.4), and the inequality follows from $U''(\Pi) < 0$. Substituting Eq. (A.6) into Eq. (A.7) yields

$$\mathrm{Cov}[U'(\tilde{\Pi}^{\diamond}), \tilde{\Pi}^{\diamond}] = \mathrm{E}[U'(\tilde{\Pi}^{\diamond})]\mathrm{Cov}(\tilde{\theta}, \tilde{S})(Z^{\diamond} - PQ^{\circ}) < 0, \tag{A.8}$$

It follows from Eq. (A.8) that $Z^{\diamond} < PQ^{\circ}$ if $\mathrm{Cov}(\tilde{\theta}, \tilde{S}) > 0$.

Evaluating the left-hand side of Equation (A.4) at $H^{\diamond} = PQ^{\circ}$, and using $U'(\Pi) = a - \Pi$ yields

$$\mathrm{Cov}\{a - \mathrm{E}(\tilde{S})PQ^{\circ} + C(Q^{\circ}) + (\tilde{\theta} - 1)\tilde{S}(Z^{\diamond} - PQ^{\circ}), \tilde{S}\}$$

$$= \mathrm{Cov}[(\tilde{\theta}-1)\tilde{S}, \tilde{S}](Z^{\diamond} - PQ^{\circ}). \tag{A.9}$$

Using Eq. (16), we have

$$\mathrm{Cov}[(\tilde{\theta}-1)\tilde{S}, \tilde{S}] = \beta \mathrm{Cov}\{[\tilde{S} - \mathrm{E}(\tilde{S})]\tilde{S}, \tilde{S}\} + \mathrm{Cov}(\tilde{\varepsilon}\tilde{S}, \tilde{S})$$

$$= \beta \mathrm{E}\{[\tilde{S} - \mathrm{E}(\tilde{S})]^2 \tilde{S}\} > 0, \tag{A.10}$$

where the second equality follows from the fact that $\tilde{\varepsilon}$ is independent of $\tilde{S}$. It then follows from Eq. (A.10) and $Z^{\diamond} < PQ^{\diamond}$ that the right-hand side of Eq. (A.8) is negative. Hence, Eq. (A.4) and the second-order conditions for program (A.2) imply that $H^{\diamond} > PQ^{\circ}$.

To sign $\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S}, \tilde{\theta}]$, program (A.2) is reformulated as a two-stage optimization problem. In the first stage, the firm takes Z as given and chooses Q and H so as to maximize its expected utility. Denote the optimal solution to the first-stage optimization problem as $Q(Z)$ and $H(Z)$. In the second stage, the firm takes $Q(Z)$ and $H(Z)$ as given and chooses Z so as to maximize its expected utility. The complete solution to program (A.2) is thus given by $Z^{\diamond}$, $Q^{\diamond} = Q(Z^{\diamond})$, and $H^{\diamond} = H(Z^{\diamond})$. Let EU be the objective function of program (A.2) with Q and H replaced by $Q(Z)$ and $H(Z)$, respectively. Totally differentiating EU with respect to Z, using the envelope theorem, and evaluating the resulting derivative at $Z = 0$ yields

$$\left.\frac{\mathrm{d}EU}{\mathrm{d}Z}\right|_{Z=0} = \mathrm{E}[U'(\tilde{\Pi}^*)(1-\tilde{\theta})\tilde{S}] = -\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S}, \tilde{\theta}], \tag{A.11}$$

which follows from the fact that $Q(0) = Q^*$ and $H(0) = H^*$. By the strict concavity of EU, it follows from Eqs. (A.5) and (A.11) that $\mathrm{Cov}[U'(\tilde{\Pi}^*)\tilde{S}, \tilde{\theta}] > 0$ if $Z^{\diamond} < 0$.

Evaluating the left-hand side of Eq. (A.5) at $Z^{\diamond} = 0$, and using $U'(\Pi) = a - \Pi$ yields

$$\mathrm{Cov}\Big\{\{a - \tilde{\theta}\tilde{S}PQ^{\circ} - [\mathrm{E}(\tilde{S}) - \tilde{S}]H^{\diamond} + C(Q^{\circ})\}\tilde{S}, \tilde{\theta}\Big\}$$

$$= \mathrm{Cov}(\tilde{\theta}, \tilde{S})[a - \mathrm{E}(\tilde{S})PQ^{\circ} + C(Q^{\circ})] - \mathrm{Cov}[(\tilde{\theta}-1)\tilde{S}^2, \tilde{\theta}]PQ^{\circ}$$

$$+\text{Cov}\{[\tilde{S}-\text{E}(\tilde{S})]\tilde{S},\tilde{\theta}\}(H^{\diamond}-PQ^{\circ}). \tag{A.12}$$

Since $\text{Cov}\{[\tilde{S}-\text{E}(\tilde{S})]\tilde{S},\tilde{\theta}\}=\beta\text{E}\{[\tilde{S}-\text{E}(\tilde{S})]^2\tilde{S}\}>0$ and $H^{\diamond}>PQ^{\circ}$, the last term on the right-hand side of Eq. (A.12) is positive. Condition (17) ensures that the sum of the first terms on the right-hand side of Eq. (A.12) is positive. It then follows from Eqs. (A.5) and (A.12) and the second-order conditions for program (A.2) that $Z^{\diamond}<0$. Hence, we have $\text{Cov}[U'(\tilde{\Pi}^*)\tilde{S},\tilde{\theta}]>0$.

Appendix B. Proof of Proposition 5

Let $F(S)$ be the cumulative distribution function of $\tilde{S}$ over support $[\underline{S},\overline{S}]$, where $0<\underline{S}<\overline{S}$. Evaluating the left-hand side of Eq. (6) at $H^*=PQ^*$ yields

$$\int_{\underline{S}}^{\overline{S}}\text{E}\Big\{U'\{\bar{\Pi}^*+\beta[S-\text{E}(\tilde{S})]SPQ^*+\tilde{\varepsilon}SPQ^*\}\Big\}[S-\text{E}(\tilde{S})]\ \text{d}F(S)$$

$$=\int_{\underline{S}}^{\overline{S}}\Big\{\text{E}\Big\{U'\{\bar{\Pi}^*+\beta[S-\text{E}(\tilde{S})]SPQ^*+\tilde{\varepsilon}SPQ^*\}\Big\}$$

$$-\text{E}[U'(\bar{\Pi}^*+\tilde{\varepsilon}SPQ^*)]\Big\}[S-\text{E}(\tilde{S})]\ \text{d}F(S)$$

$$+\int_{\underline{S}}^{\overline{S}}\Big\{\text{E}[U'(\bar{\Pi}^*+\tilde{\varepsilon}SPQ^*)]-\text{E}\{U'[\bar{\Pi}^*+\tilde{\varepsilon}\text{E}(\tilde{S})PQ^*]\}\Big\}[S-\text{E}(\tilde{S})]\ \text{d}F(S), \tag{A.13}$$

where $\bar{\Pi}^*=\text{E}(\tilde{S})PQ^*-C(Q^*)$. Differentiating $\text{E}[U'(\bar{\Pi}^*+\tilde{\varepsilon}SPQ^*)]$ with respect to S yields

$$\frac{\partial\text{E}[U'(\bar{\Pi}^*+\tilde{\varepsilon}SPQ^*)]}{\partial S}=\text{E}[U''(\bar{\Pi}^*+\tilde{\varepsilon}SPQ^*)\tilde{\varepsilon}]PQ^*$$

$$=\text{Cov}[U''(\bar{\Pi}^*+\tilde{\varepsilon}SPQ^*),\tilde{\varepsilon}]PQ^*>0, \tag{A.14}$$

where the inequality follows from $U'''(\Pi)>0$. Hence, it follows from Eq. (A.14) that the second term on the right-hand side of Eq. (A.13) is positive given prudence. If $\beta=0$, the first term on the right-hand side of Eq. (A.13) vanishes. It then follows from Eqs. (6) and (A.13) and the second-order conditions for program (2) that $H^* < PQ^*$ if $\beta = 0$. On the other hand, if $\beta < 0$, it follows from $U''(\Pi) < 0$ that

$$U'\{\bar{\Pi}^* + \beta[S - \mathrm{E}(\tilde{S})]SPQ^* + \varepsilon SPQ^*\} < (>)\ U'(\bar{\Pi}^* + \varepsilon SPQ^*), \tag{A.15}$$

for all $S < (>)\ \mathrm{E}(\tilde{S})$. Taking expectations on both sides of Eq. (A.15) with respect to the random variable, $\tilde{\varepsilon}$, yields

$$\mathrm{E}\Big\{U'\{\bar{\Pi}^* + \beta[S - \mathrm{E}(\tilde{S})]SPQ^* + \tilde{\varepsilon} SPQ^*\}\Big\} < (>)\ \mathrm{E}[U'(\bar{\Pi}^* + \tilde{\varepsilon} SPQ^*)], \tag{A.16}$$

for all $S < (>)\ \mathrm{E}(\tilde{S})$. Hence, it follows from Eq. (A.16) that the first term on the right-hand side of Eq. (A.13) is positive if $\beta < 0$. Eqs. (6) and (A.13) and the second-order conditions for program (2) then imply that $H^* < PQ^*$ if $\beta < 0$.

References

Adam-Müller, A. F. A. (1997). Export and hedging decisions under revenue and exchange rate risk: a note. European Economic Review, 41, 1421–1426.

Benninga, S., Eldor, R., & Zilcha, I. (1985). Optimal international hedging and output policies in commodity and currency forward markets. Journal of International Money and Finance, 4, 537–552.

Briys, E., Crouhy, M., & Schlesinger, H. (1993). Optimal hedging in a futures market with background noise and basis risk. European Economic Review, 37, 949–960.

Broll, U., & Zilcha, I. (1992). Exchange rate uncertainty, futures markets and the multinational firm. European Economic Review, 36, 815–826.

Drèze, J. H., & Modigliani, F. (1972). Consumption decisions under uncertainty. Journal of Economic Theory, 5, 308–335.

Eeckhoudt, L., & Schlesinger, H. (2006). Putting risk in its proper place. American Economic Review, 96, 280–289.

Gollier, C. (2001). The Economics of Risk and Time. Cambridge, MA: MIT Press.

Katz, E., & Paroush, J. (1979). The effect of forward markets on exporting firms. Economics Letters, 4, 272–274.

Kawai, M., & Zilcha, I. (1986). International trade with forward-futures markets under exchange rate and price uncertainty. Journal of International Economics, 20, 83–98.

Kim, J.-C., & Song, K. (2006). Country risk, fund characteristics, and premiums on closed-end country funds. Working paper, Sungkyunkwan University, Seoul, South Korea.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. Econometrica, 58, 53–73.

Kimball, M. S. (1993). Standard risk aversion. Econometrica, 61, 589–611.

Leland, H. E. (1968). Saving and uncertainty: The precautionary demand for saving. Quarterly Journal of Economics, 82, 465–473.

Mossin, J. (1968). Aspects of rational insurance purchasing. Journal of Political Economy, 76, 553–568.

Viaene, J.-M., & Zilcha, I. (1998). The behavior of competitive exporting firms under multiple uncertainty. International Economic Review, 39, 591–609.

Wong, K. P. (2003a). Export flexibility and currency hedging. International Economic Review, 44, 1295–1312.

Wong, K. P. (2003b). Currency hedging with options and futures. European Economic Review, 47, 833–839.