Selective Disclosure of Public Information: Who Needs to Know?

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March 2010

Abstract

Credibly communicating information for agents to act upon is a challenge for central planners and managers. This paper shows that in settings where there are strategic complementarities or substitutability among agents that the accuracy of the public announcement may only be communicated by denying some agents' access to the information. When agents' actions are complementary, the most accurate information is rationed to signal the value of the announcement. In contrast when there is interference between agents' actions, it is the least accurate information that is selectively disclosed. The rationale for this pattern of selective disclosure is best understood as a response to agents' misuse or misinterpretation of public announcements in different strategic settings. This paper illustrates how public information is best controlled and rationed with some applications to controlling congestion, beauty contests, macroecomic stablilization and promoting uniformity and standardization in design and adoption.

1 Introduction

Much public and industrial policy is directed towards providing private agents with public information to enable individuals to act efficiently and to optimally coordinate their actions. In almost every private and public sector, including financial markets, manufacturing, housing, medical care and the environment, public or centrally supplied information on the "state of the world" directs agents to act as well as to form expectations about the behavior of other agents. While we have relatively well developed models predicting how agents optimally interpret and react to public information, little attention has been directed to the issue of how public information is credibly conveyed to individual decision makers. This is the topic we address in this paper.

To motivate our analysis consider this general setting which has been extensively studied in a variety of different contexts. A large group of agents, working for themselves or for an organization, decide how and in what way to direct their personal effort. The agents each receive independent private and normally distributed information about their environment upon which to act. The aggregate surplus the agents generate is quadratic in the agents actions and the state and it depends on two factors: (i) the individual output derived from each agent's effort and (ii) the joint output resulting from the interaction of the agents' different actions. The joint surplus captures strategic complementarities or substitutability resulting when agents' actions aren't coordinated. Working in isolation, each agent selects effort to maximizes his personal surplus based on his private knowledge of the environment. Unfortunately, the aggregate surplus derived from the agents' individual actions is invariably suboptimal. Agents underperform when acting in isolation because they have incomplete private information to act on and because their behavior is not coordinated.

Aggregate performance may be improved by a benevolent planner or manager that provides agents with more precise public information about their underlying state upon which to act—-or so it would seem. But herein lie two challenges inherent with supplying public information The first is how should the agents optimally react to and process the public information? And second, how can the planner credibly communicate the reliability of the infomation, when the accuracy of the announcement can not be objectively verified? Our analysis attempts to study these issues in a simple, yet relatistic, model where the manager may only affect agents' actions by her disclosure of information among different groups.

The answer to the first question of the agents' optimal reaction to public information turns on how the planner wishes to direct the agents' effort. Angeletos and Pavan (2007) and Hellwig (2005) show that when agents' efforts are complementary social welfare is increased if the agents pay greater attention to public information. The opposite is true when agents' effort are substitutes. We show this leads to a *pursuasion motive* for the planner that dictates how she discloses public information. Specifically, when there are strategic complementarities the planner wishes to signal that public information is of the highest accuracy to pursuade agents to pay more attention to it. In contrast the planner signals that public information is of the lowest percision to pursuade agents to pay less attention to public disclosure, when agents' actions are substitutes.

To answer the second question of how the accuracy of public information is credibly comunicated, again requires one to consider the planner's motives for disclosing information. Recall, the planner can not directly demonstrate the reliability of her public disclosure, whether it is high or low, because the accuracy is not publicly observed. Instead the planner exploits preference differences for disclosing information that exist when planners with high versus low accuracy information signal their type. Since the relative preferences for disclosure among high and low types varies by the planner's *pursuasion motive*, we expect to find varying signals of the reliability of public information in different environments.

To illustrate, consider the complementarity environment, where agents pay insufficient attention to public disclosure. Both the high and low precision type planner wish to pursuade agents their information is precise to induce greater conformity in agents' effort. Given the value of information for decision making, neither type planner wants to prevent any agent from becoming better informed. However planners of either type would consider rationing information to a subset of agents, to pursuade the other, non-rationed, agents of the importance of her public disclosure. In what follows we demonstrate the cost of pursuasion– that is the foregone social suplus from rationing–is lower for the high type than for the low type planner. We show this implies a signaling equilibirum exists where high type planners ration information to certain agents to signal the reliability of their disclosure, whereas information is provided to all agents when the disclosure is less accurate. Indeed this is the unique equilibrium for reasonable out of equilibrium beliefs (e.g.beliefs satisfying the intuitive criterion). The irony that public disclosure is rationed precisely when it is most informative, reflects the planner's cost of credibly communicating the reliability of her information.

Signaling the reliability of public disclosure when agents' effort are substitutes, is the symmetric opposite to the strategic complements setting. Consider the "beauty contest" setting of Morris and Shin (2002) where agents focus excessively on public disclosure to predict other agents' behavior. Now the planner tries to pursuade agents her information is less accurate to induce them to act more independently. We demonstrate the cost of pursuasion is lower for the low type than for the high type planner. Hence, in equilibirum information is rationed when it is least accurate and it is widely disseminated when it is most relaible. Recall, that agents' focus on public disclosure is welfare decreasing, so now the irony is information is most available precisely, when it is most accurate but least valuable for raising social surplus.

The analyses summarized above provide some insights regarding how and why public information is disclosed to varying degrees with respect to different public issues and debates ranging from economic stabilization, to welfare reform to environmental policy. We conclude the paper with some applications of our findings to different settings. For instance when public announcements are useful in managing the real effects and expected responses to supply market shocks how is public information disclosed? What group (or "inner circle"—if you will) does the planner inform? We find, somewhat to our surprise, that it is not necessarily the most receptive or easily influenced population who is informed, but rather it is the group that minimizes the planner's relative cost of signaling. We illustrate this implies that agents receive public disclosures based on the accuracy of their private information. This means that public disclosures are targeted towards the less informed agents, in environments where greater synchronization of actions is desired. The opposite is true,—- it is the more informed agents who are selectively briefed— when greater response to variations in underlying fundamentals of the market is needed.

Finally we inquire about the desirability of *transparency* in public disclosure. When pubic information is known to increases social welfare, transparency must be unambigously preferred—or is it? What if, as we assume, the reliability of the public disclosure is known only by the planner? Is transparency, that is a commitment to disclose public information to all agents, preferable to *discretionary disclosure* of information to selected agents? The latter policy enables the planner to signal the reliability of the information, albeit at the cost of denying access to some agents, whereas the former policy insures full disclosure of *all* information regardless of its accuracy. We demonstrate that (results yet to come).

Our plan for the rest of the paper is as follows. Section 2 contains our model, describes the role of the central planner in managing information and compares our approach to analyzing public disclosure with other previous studies in the literature. We demonstrate there is a unique signaling equilibrium and characterize the equilibrium disclosure policy and agents' responses in Section 3. Application and implications of our findings for a variety of public and managerial settings are examined in Section 4. Section 5 concludes with a summary of results and a description of some open questions for future research. The appendix contains proofs of formal results that do not appear in the body of the paper.

2 Public Disclosure of Information of Known Accuracy

When is public information beneficial and how is it best disclosed? The natural place to begin to address this question is for the canonical setting, analyzed in literature, where the reliability of public disclosure is common knowledge.

2.1 Decision Makers

Consider an economy or organization consisting of a continuum of measure of one of decision makers indexed by i uniformly distributed over $[0,1]$. For simplicity we adopt a standard reduced form specification due to Angeletos and Pavan (04) to model agent's preferences in which each agent i is risk neutral with utility,

$$U_i^a = Ae_i - \frac{1}{2}e_i^2 \tag{1}$$

Each agent i's surplus,U_i^a is a function of his choice of action (or effort),$e_i \in \mathbb{R}$, and A which represents the aggregate return to effort. We assume that $\bar{e} = \int_0^1 e_i di$ measures the aggregate (or mean) effort allocated and that A is determined by,

$$A = (1-\rho)\, v + \rho\bar{e} \tag{2}$$

where v is a random state (reflecting underlying conditions) and $\rho\bar{e}$ measures the impact of other agent's actions on the individual returns to agent i from his effort. The sign of the coefficent ρ reflects whether agents' efforts are complements, as when $\rho > 0$ or substitutes when $\rho < 0$.

It is illustrative to rewrite the agent's objective function as

$$\begin{aligned} U_i^a &= [(1-\rho)\,v + \rho\bar{e}]\,e_i - \frac{1}{2}e_i^2 \\ &= \underbrace{-\frac{1}{2}(1-\rho)(e_i - v)^2}_{\text{"Better Action"}} \underbrace{-\frac{1}{2}\rho\,(e_i - \bar{e})^2}_{\text{"Coordination"}} + \frac{1}{2}\rho\bar{e}^2 + \frac{1}{2}(1-\rho)\,v^2. \end{aligned} \tag{3}$$

Because agents take $\bar{e}$ as given, (3) shows that agents choose effort to maximize a weighted average of two objectives: better action $(e_i - v)^2$ and coordination with other agents $(e_i - \bar{e})^2$. When $\rho > 0$, agents benefit when their actions are closer to each other. When $\rho < 0$, agents would be better off when their actions are further apart.

The above setting might be one in which independent agents in the economy decide on how much to invest or how much effort to expend to maximize their individual expected wealth. Alternatively, the agents might represent separate divisions of a firm that are paid according to the surplus they independently generate. In either case the individual actions of different agents may strategicly complement or subsitutute for each other in the aggregate.

2.2 Social Planner: Preferences, Information and Disclosure Policy

A benevolent social planner, P, exists to help manage the agents' behavior to maximize social surplus. Social surplus, defined by $U^P \equiv \int_0^1 U_i^a di$, is determined by (1) and (2) as,

$$U^P = A\bar{e} - \frac{1}{2}\int_0^1 e_i^2 = (1-\rho)\,v\bar{e} - (1-2\rho)\,\frac{1}{2}\bar{e}^2 - \frac{1}{2}\int_0^1 (e_i - \bar{e})^2\,di$$

We assume $\rho < 1/2$ to insure U^P is concave in $\bar{e}$ and thereby well behaved.

We can similarly rewrite the social surplus as:

$$U^P(v) = \underbrace{-\frac{1}{2}(1-\rho)\int E\,(e_i - v)^2\,di}_{\text{"Better Action"}} \underbrace{-\frac{1}{2}\rho\int E\,(e_i - \bar{e})^2\,di}_{\text{"Coordination"}} + \frac{1}{2}\rho E\,(\bar{e})^2 + \frac{1}{2}(1-\rho)\,v^2.$$

Thus, like the agents, the social planner cares about the agents taking the right actions as well as coordination. In addition, the social planner also cares about $\frac{1}{2}\rho\bar{e}^2$ on the aggregate.

The random state of nature is unknown prior to each agent's effort selection. For simplicity, agents share a common prior that v is uniformly distributed on $\mathbb{R}$. Prior to choosing e_i agent i receives a private, noisy signal s_i of v:

$$s_i = v + \varepsilon_i,$$

where $\varepsilon_i \sim N(0, 1/\beta_i)$ and $cov(\varepsilon_i, v) = cov(\varepsilon_i, \varepsilon_k) = 0$, $\forall i \neq k$. The precision of $i's$ signal is (for now) the same for all agents with $\beta_i = \beta$ for all i. The planner also observes a signal, z of v

$$z = (v + \eta)$$

where $\eta \sim N(0, 1/\alpha)$,and $cov(v, \eta) = cov(\varepsilon_i, \eta) = 0$, $\forall i$.

Departing from previous models of public disclosure we assume the absolute precision of either the public or private signal is not observed by the agents. However, for the present we assume the *relative precision*s of the public and private signal denoted by $\lambda \equiv \alpha/\beta$, is common knowledge, This assumption is without loss of generality here. However, later when we assume that agents are unable even to observe relative accuracies, this assumption, that the absolute precision of private information is unknown, permits us to focus attention on disclosure policy as a signal for the relative accuracy of public information.

2.2.1 Disclosure Policy

The planner serves as a faithful agent for the decision makers. Whatever public information she discloses is truthful; she can not transmit false or distorted information. The planner may only influence the action each agent selects and thereby the social surplus that is generated by the *transparency* of her public disclosure. The planner discloses to each agent i a signal, z_i, given by

$$z_i = \delta_i z + (1 - \delta_i)\emptyset; \;\; \delta_i \in \{0, 1\}$$

According to () the agent observes the informative public signal z when $\delta_i = 1$, otherwise if $\delta_i = 0$, he receives the *null* signal, $\emptyset$, that contains no information.

Prior to observing z, the planner commits to a *disclosure policy* DP which is common knowledge. DP is a selection of a subset $D \subseteq [0,1]$ of agents i who are publicly informed with $(\delta_i = 1)$. Let $\bar{\delta} = \int_{i \in D} \delta_i di$ be the measure of D. We refer to D as the planner's "inner circle" who are privy to her information. A completely *transparent* DP is one where $\bar{\delta} = 1$ and $D = [0,1]$, whereas an *opaque* DP is one where the inner circle is a "proper" subset of the agent population, $D \subset [0,1]$. Finally, communication among agents is ruled out so that an agent $j \notin D$ may not learn the content of the public signal or infer anything about the pubic signal since D is selected prior to the realization of z.

2.3 Equilibrium

For a given disclosure policy DP and realization of private and public signals, each agent updates his beliefs about the state of nature and behavior of other agents from his information set $\Sigma_i = \left\{ s_i, z_i \mid \lambda, \bar{\delta} \right\}$. The posterior of i is that state v, is normally distributed with mean

$$E_i \left(v \mid \Sigma_i \right) = \delta_i \gamma z + \left(1 - \delta_i \gamma \right) s_i$$

where:$\gamma = \lambda / \left(1 + \lambda \right)$. Notice an agent j otuside the *inner circle*, with $\delta_j = 0$ updates based only on his private information.

Agents i simultaneously chooses e_i to maximze $E_i \left[U_i^a \right]$ which requires

$$e_i = E_i \left[A \right] = \left(1 - \rho \right) E_i \left[v \right] + \rho E_i \left[\bar{e} \right]$$

where $E_i \left[\bar{e} \right]$ is agent $i's$ expectation of the other agents' aggregate effort. As expected, effort is increasing in the underlying state and it is increasing in expected aggregate effort when agents' efforts are strategic complements as for $\rho > 0$. Otherwise effort is decreasing in $\bar{e}$ when efforts are strategic substitutes as for $\rho < 0$.

Given that the equation for e_i is linear in the updated beliefs about v which are normal, it's reasonable to search for an equilibrium among the set of effort supply functions that are linear in the private and public signals. Indeed, following Pavan and D'Angelitos we are able to show there is a supply equilibirum that is linear and that it is the unique equilibrium.

Proposition 1 *A unique effort supply equilibrium exists, given by* $e_i = \delta_i w z + \left(1 - \delta_i w \right) s_i$ *where*

$$w = \frac{\lambda}{\lambda + 1 - \rho \bar{\delta}} \tag{4}$$

with

$$e_i = \begin{cases} s_i & for \quad i \notin D \\ e_i = wz + (1-w)\, s_i & for \quad i \in D \end{cases}$$

Proof : The proof is similar to Angeletos and Pavan (2004) and is therefore ommitted.

2.4 Welfare and Transparency

We are now ready to address the original question that motivated our analysis: *when is public information beneficial and how is it best disclosed?* For a given DP with an inner circle of size $\bar{\delta} \in [0,1]$, the social surplus in equilibrium is given by:

$$\begin{aligned} U^P(v) &= \frac{v^2}{2} - \frac{L(\bar{\delta})}{2} \\ \text{where} \quad L(\bar{\delta}) &= \frac{1}{\beta}\left(\frac{\bar{\delta}\left(1-2\rho\bar{\delta}\right)w^2}{\lambda} + \bar{\delta}(1-w)^2 + 1 - \bar{\delta}\right), \end{aligned}$$

where w is the weight agents place on the public signal.

The goal for the social planner is to implement an information acquisition and management policy to minimize these losses. To see the role of disclosure policy, we begin by identifying the inefficiency in agents' equilibrium effort choices. The inefficiency is expected because agents do not take full consideration of the impact of their individual actions on the aggretate level. As a result, their use of information is inefficient relative to the social optimal level.

To see this more clearly, suppose the planner could direct agents on how to weigh pubic and private information. She would choose w as

$$w^{FB} = \arg\min_{w} L\left(w|\bar{\delta}\right) = \frac{\lambda}{\lambda + 1 - 2\rho\bar{\delta}}. \tag{5}$$

Compare (5) with (4),we have

$$\frac{w^{FB} - w^*}{w^{FB}} = \frac{\rho\bar{\delta}}{\left(\lambda + 1 - \rho\bar{\delta}\right)}. \tag{6}$$

Therefore, when $\rho > 0$, agents under-weight public information, and this happens despite the fact that agents already put hightened attention to public signal when their actions are strategic complements. The opposite is true when $\rho < 0$: relative to the socially optimal level, agents over-weight public signal when their actions are strategic substitutes.

It is obvious from (6) that more precise public information (higher λ) moves the w^* closer to w^{FB} and therefore improves social welfare. This is further verified as

$$\begin{aligned} \frac{\partial U^P(v)}{\partial \lambda} &= w\left(1+\frac{1}{\lambda}\right)\frac{\partial w}{\partial \lambda} + w\frac{\partial w\left(1+\frac{1}{\lambda}\right)}{\partial \lambda} \\ &= {}_s\left(1+\frac{1}{\lambda}\right)\frac{1-\rho\bar{\delta}}{\left(\lambda+1-\rho\bar{\delta}\right)^2} + \frac{\rho\bar{\delta}}{\left(\lambda+1-\rho\bar{\delta}\right)^2} \\ &= {}_s\left(1+\frac{1}{\lambda}\right)\left(1-\rho\bar{\delta}\right)+\rho\bar{\delta} > 0\ \forall \rho < 1/2. \end{aligned}$$

The general intuition is that public information always help agents achieve better action to match the underlying state of the world. Further, when there is strategic externality in agents' actions, public information is more important than private information in forecasting the aggregate effort. When agents care about how their actions relate to each other (either complementarity or substitution), the better ability to forecast aggregate effort is always welfare improving.

The arrival of more accurate public information is usually accompanied by increased demand for transparency. A more transparent policy informs a greater number of agents who then harness their knowledge to make better decisions– or so it would seem. But is transparency welfare increasing in all settings, even those where public information may be unreliable? Conditions for full disclosure to be optimal for any information structure are recorded in the following:

Proposition 2 *Social welfare is maximized at a full disclosure DP with* $\bar{\delta} = 1$ *provided that* $\rho \geq \underline{\rho}$ *where* $\underline{\rho}(\lambda) < 0$ *and* $\underline{\rho}'(\lambda) < 0$.

When $\rho > 0$, higher $\bar{\delta}$ unambiguously increases social welfare for any λ. This is because it provides more agents valuable public information for better decision making. Further, it reduces the inefficiency embeded in agents' equilibrium weight, as

$$\frac{\partial\left(\frac{w^{FB}-w^*}{w^{FB}}\right)}{\partial \bar{\delta}} = \rho\frac{\lambda+1}{\left(\lambda+1-\rho\bar{\delta}\right)} > 0 \text{ iff } \rho > 0. \tag{7}$$

Recall agents under-weight public information when $\rho > 0$, (7) implies that more transparent disclosure policy (larger $\bar{\delta}$) reduces inefficient weighting thus improving welfare.

However, when $\rho < 0$, agents over-weight public information. (7) implies that larger $\bar{\delta}$ magnifies the weighting inefficiency, more so when ρ is more negative. Thus, full disclosure is desirable only when the benefit of providing more agents with valuable decision making information is high, which is the case when λ is high enough.

The above analysis implies that pubic information and disclosure are complements. Full disclosure is desirable as long as there is the degree of substitution is not too high. Alternatively, because $\underline{\rho}'(\lambda) < 0$, it also implies that for any ρ, full disclosure is always beneficial when λ is high enough. The sections to follow examine the extent to which this is a general feature that holds in environments where the accuracy of public information is hard to observe.

3 Public Information Disclosure of Unverifed Accuracy

Having access to more accurate information is beneficial, at least for individual decision makers. And, as the previous section illustrates, this conclusion even extends to social situations where individuals' decision may impact positively or negatively on each other. But what is not known is whether access to more precise information is individually and socially beneficial, when the accuracy of the information itself is not observed by all. This is the challenge for an agent who must decide how to process and react to public information of unknown accuracy.

3.1 Types of Pubic Information, Disclosure and Agents Response

To address this issue consider the following modification to our model of public disclosure. Suppose there are two rather than one type of public signal(s) the planner may receive; one is a (h) type, *high relative precision* signal and the other is a (l) type, *low relative precision* signal. Assume the relative precisions of the l and h type signals are ordered so that,

$$1 \leq \lambda_l < \lambda_h$$

and that q is the ex ante probability the signal is high precision. Ex post, while the planner knows the relative accuracy of the public signal, agents are unable to observe the signal type whether it is h or l. Moreover, without knowing the precision of their own private information, agents' are unable to update their beliefs of the accuracy of public disclosure by comparing it with their private information.

Although the planner's information type can not be observed, agents may infer the relative precision of public information indirectly by observing the planner's disclosure policy. Recall that a disclosure policy is a selection of agents $D \subseteq [0,1]$ who are publicly informed. The planner with a certain *type* of information may signal the informativeness of his signal by the transparency of his disclosure. For instance a *high type* planner may wish to pursuade agents that his information is very accurate by restricting the disclosure of the public signal to a select set of agents. Since all agents receive the same accuracy private information, we may without loss of generality restrict attention to the set of disclosure policies

$$DP : \Lambda \rightarrow \bar{\delta} \in [0,1]$$

where DP is a mapping from the set of all precision types $\Lambda = \{\lambda_i\}_{i=l,h}$ into the fraction, $\bar{\delta}$, of agents that are publicly informed.

Upon observing the planner's disclosure policy, agents may infer what type(s) the planner may be. Let $q\left(\bar{\delta}\right)$ be the agent's posterior probability the planner is of type h after observing $\bar{\delta}$. Upon observing $\bar{\delta}$ and her private and public signals, (s_i, z_i) each agent i simultaneously selects an e_i given her information set $\Sigma_i = \left\{s_i, z_i \mid q\left(\bar{\delta}\right)\right\}$ in order to her expected utility given by

$$\max_{e_i} q\left(\bar{\delta}\right) E\left[Ae_i - \frac{1}{2}e_i^2|\lambda_h\right] + \left(1 - q\left(\bar{\delta}\right)\right) E\left[Ae_i - \frac{1}{2}e_i^2|\lambda_l\right]$$

There is a unique effort supply equilibrium resulting that is characterized by

LEMMA 1: *A unique effort supply equilibrium exists, given by*

$$e_i = \delta_i w\left(q\left(\bar{\delta}\right)\right) z + \left(1 - \delta_i w\left(q\left(\bar{\delta}\right)\right)\right) s_i$$

where

$$w\left(q\left(\bar{\delta}\right)\right) = q\left(\bar{\delta}\right) \frac{\lambda_h}{\lambda_h + 1 - \rho\bar{\delta}} + \left(1 - q\left(\bar{\delta}\right)\right) \frac{\lambda_l}{\lambda_l + 1 - \rho\bar{\delta}}$$

with

$$e_i = \begin{cases} s_i & \text{for } \; i \notin D \\ e_i = w\left(q\left(\bar{\delta}\right)\right) z + \left(1 - w\left(q\left(\bar{\delta}\right)\right)\right) s_i & \text{for } \; i \in D \end{cases}$$

3.2 Planner's Payoffs and Signalling Preferences:

Although each type planner wishes to maximize expected social surplus, planners with different precision public information derive different surplus from different disclosure policies. In equilibrium a particular planner of relative accuracy λ with disclosure policy $\bar{\delta}(\lambda)$ derives social surplus given by

$$U^P\left(\lambda, \hat{\lambda}\left(\bar{\delta}\right), \bar{\delta}\right) \equiv \frac{v^2}{2} - \frac{L\left(\lambda, \hat{\lambda}\left(\bar{\delta}\right), \bar{\delta}\right)}{2}$$

$$\text{where} \quad L\left(\lambda, \hat{\lambda}\left(\bar{\delta}\right), \bar{\delta}\right) = \frac{\left(\bar{\delta}\left(1-2\rho\bar{\delta}\right) w^2\left(\hat{\lambda}\left(\bar{\delta}\right)\right) + \bar{\delta}\lambda\left((1-w\left(\hat{\lambda}\left(\bar{\delta}\right)\right))^2 + \left(1-\bar{\delta}\right)\right)\right)}{\alpha}$$

$$w\left(\hat{\lambda}\left(\bar{\delta}\right)\right) = \frac{\hat{\lambda}\left(\bar{\delta}\right)}{\hat{\lambda}\left(\bar{\delta}\right) + \left(1-\rho\bar{\delta}\right)}$$

It's clear from inspection, that the social surplus from any disclosure policy depends on the agent's beliefs, $\hat{\lambda}\left(\bar{\delta}\right)$, regarding the precision type of public informtion conditional on the disclosure policy, $\bar{\delta}$. The planner may try to pursuade agents that public information is relatively accurate or inaccurate by the disclosure policy he selects. A first step in predicting how the planner strategically chooses his disclosure policy therefore is to know what he would prefer the agents to believe about the public signal precision. The following lemma characterizes conditions under which the planner would claim that his disclosure is of high or low accuracy to maximize social surplus when disclosing public information to the entire population of agents.

LEMMA 2: *When $\rho > 0$ the high type always reports his true accuracy and the low type misreports her accuracy as being high provided $\frac{\lambda_l}{\lambda_h} \geq \frac{\left(1-2\rho\bar{\delta}\right)}{\left(1-\rho\bar{\delta}\right)}$. When $\rho < 0$ the low type always reports his true accuracy and the high type misreports his accuracy is low provided $\frac{\lambda_l}{\lambda_h} \leq \frac{\left(1-\rho\bar{\delta}\right)}{\left(1-2\rho\bar{\delta}\right)}$.*

Planners are often *parental* in their disclosure of information and advice, hoping to guide the agents they oversee to make the best decision for their own collective good. Our planner is no exception in this regard. When the agent's individual efforts are strategic complements the planner wishes the agents to better coordinate their effort to maximize collective surplus. If the agents believe that his information is highly accurate, they would pay more attention to the public signal when choosing effort. This results in a higher correlation of independent effort allocations leading to greater social surplus. In contrast when agents efforts are strategic substitutes, the planner wishes to pursuade the agents that his disclosure is of relatively low accuracy. The rationale here is that agents are induced to differentiate their effort choices if they believe the public signal is less accurate, and they pay less attention to it in selecting effort. As a result there is less correlation in individual effort choices which leads to greater social surplus.

3.3 The Signaling Game with Unverifiable Accuracy

Agents' perception of the relative accuracy of public disclosure is informed by observing the disclosure policy that the planner selects. As Proposition 2 shows, planners of all types would prefer a transparent full disclosure policy, where it not for the fact that agents reaction to public information depends on how widely it is disclosed. This process induces a signaling game which unfolds in the following sequence. In the *first stage* after the planner observes the accuracy of public information, she then selects a disclosure policy, $\bar{\delta} \in [0,1]$, that is pubicly observed by all agents, who then update their beliefs about public information accuracy from the disclosure policy. Next, in *stage two*, the planner and each agent observe their respective information. Following in *stage three*, the public information is disclosed by the planner to the agents in D. The game concludes in *stage four* where alll agents simultaneously select effort based on their updated information to maximize expected surplus. The analysis that follows below characterizes the signaling equilibrium (Perfect Bayesian Equilibirum) for this game.

In order to characterize the equilibrium disclosure policies of different planners, it is useful to determine which types of planners benefit most from pursuasion of agents. A first step in that direction is to verify that single crossing properties holdfor our model.

LEMMA 3: $\frac{d}{d\lambda}\left(-\frac{U^P_{\bar{\delta}}}{U^P_{\hat{\lambda}}}\right) = \frac{d}{d\lambda}\left(\frac{d\hat{\lambda}}{d\bar{\delta}}\right)\Big|_{U^P} > 0$

PROOF: Differentiating the planner's expected surplus, we obtain

$$U^P_{\bar{\delta}} = \frac{2\bar{\delta}\frac{dw}{d\bar{\delta}}\left(\left(1-\rho\bar{\delta}\right)-\left(1-2\rho\bar{\delta}\right)\hat{\lambda}/\lambda\right)}{\left(\hat{\lambda}+\left(1-\rho\bar{\delta}\right)\right)B} - \frac{\left(1-4\rho\bar{\delta}\right)w^2}{\alpha} - \frac{\left(1-w\right)^2-1}{\beta}$$

$$U^P_{\hat{\lambda}} = 2\bar{\delta}\left(\frac{\left(1-\rho\bar{\delta}\right)-\hat{\lambda}/\lambda\left(1-2\rho\bar{\delta}\right)}{\hat{\lambda}+\left(1-\rho\bar{\delta}\right)}\right)\frac{dw}{d\hat{\lambda}}\frac{1}{\beta}$$

where:

$$\frac{dw\left(\hat{\lambda}\right)}{d\bar{\delta}} = \frac{\hat{\lambda}\rho}{\left(\hat{\lambda}+\left(1-\rho\bar{\delta}\right)\right)^{2}}$$

$$\frac{dw}{d\hat{\lambda}} = \frac{\left(1-\rho\bar{\delta}\right)}{\left(\hat{\lambda}+\left(1-\rho\bar{\delta}\right)\right)^{2}}$$

Combining () and () and to form $\left(-\frac{U^P_{\bar{\delta}}}{U^P_{\hat{\lambda}}}\right)$ and differentiating with respect to λ we obtain

$$\begin{aligned}
\frac{d}{d\lambda}\left(-\frac{U^P_{\bar{\delta}}}{U^P_{\hat{\lambda}}}\right) &= \frac{d}{d\lambda}\left(-\frac{\frac{dw}{d\bar{\delta}}}{\frac{dw}{d\hat{\lambda}}}+\frac{\left(\frac{\left(1-4\rho\bar{\delta}\right)w^2}{\lambda}+(1-w)^2-1\right)}{2\bar{\delta}\left(-\frac{\left(1-2\rho\bar{\delta}\right)w}{\lambda}+(1-w)\right)\frac{dw}{dq}}\right) \\
&= {}^{s}-\frac{\left(1-4\rho\bar{\delta}\right)w^2}{\lambda^2}\left(\left(-\frac{\left(1-2\rho\bar{\delta}\right)w}{\lambda}+(1-w)\right)\right) \\
&\quad -\frac{\left(1-2\rho\bar{\delta}\right)w}{\lambda^2}\left(\frac{\left(1-4\rho\bar{\delta}\right)w^2}{\lambda}+(1-w)^2-1\right) \\
&= {}^{s}-\frac{(1-2\rho)w}{\lambda^2}\left((1-w)w+(1-w)^2-1\right)-\frac{(1-2\rho)w}{\lambda^2}\left((1-w)^2-1\right) \\
&= {}^{s}-\frac{(1-2\rho)w}{\lambda^2}\left((1-w)(w+1-w)-1\right) \\
&= {}^{s}\frac{(1-2\rho)w^2}{\lambda^2}>0
\end{aligned}$$

■

Confirmation of the single crossing property is all that we need to construct the unique signaling equilibrium. First consider the settng where agents efforts are complements, the case which is depicted in Figure 1a. The indifference surfaces for types h and l are represented by the curves labeled I_h and I_l which plot the combinations of disclosure policies and perceived types $\left(\bar{\delta},\hat{\lambda}\right)$, that yield constant expected surplus to the planner of type h and type l respectively. The indifference curves are downward sloping with I_h flatter than I_l for all $\left(\rho,\bar{\delta}\right)\in\Gamma_l$. *(assuming* $\hat{\lambda}$ is the $y-$axle).

There is a continuum of separating equilibrium represented by the set $\left\{\bar{\delta}^h\in[\bar{\delta},\bar{\delta}_l(\lambda_h)],\bar{\delta}_l=1\right\}$, wherein P^h selects $\bar{\delta}^h$ and P^l selects $\bar{\delta}_l=1$. There is a Pareto dominant equilibrium,

$\left\{\bar{\delta}^h = \bar{\delta}_l(\lambda_h), \bar{\delta}_l = 1\right\}$, that minimizes $P^{h\prime}s$ cost of signaling. This is also the only equilibrium that is supported by Intuitive Criterion stated below.

> **Cho and Kreps Intuitive Criterion** *Consider a type $j \in \{h, l\}$ who makes an out of equilibrium selection of $\tilde{\delta}$. Suppose her type is correctly perceived and as a result type j is better off. Then if no other type $j' \in \{h, l\}$ is better off mimicing type j, the perception of the agents is "credible".*

Looking at Figure 1a, it is easy to see why the pareto dominant equilibrium is the only equilibrium satisfying the intuitive criterion. For any other separating equilibrium, such as $\left(\bar{\delta}_h < \bar{\delta}_l(\lambda_h), \bar{\delta}_l = 1\right)$ P^h can profit by increasing $\bar{\delta}_h$ provided his type is correctly perceived. In this case there would be no reason for P^l to mimic this deviation, since he can not increase his surplus by doing so. Hence that equilibrium is eliminated by the intuitive criterion.

Pooling (or semi pooling) equilibrium also exist, where both types select the same disclosure policy with strictly positive probability. However once again these equilibrium are elimnated by the intutive criteria. To illustrate, consider the pure pooling equilibrium where $\left\{\bar{\delta}_h = \bar{\delta}^p, \bar{\delta}_l = \bar{\delta}^p\right\}$ and q is the ex ante probability $\lambda = \lambda_h$. Define $\bar{\delta}_l\left(\bar{\delta}^p\right)$ to be the disclosure policy which leaves P^l indifferent to pooling or separating by selecting an off equilibrium path disclosure $\bar{\delta}_l\left(\bar{\delta}^p\right)$ such that $U^{P^h}\left(\lambda_l, \lambda_h, \bar{\delta}_l\left(\bar{\delta}^p\right)\right) = U^{P^l}\left(\lambda_l, q, \bar{\delta}_l\left(\bar{\delta}^p\right)\right)$. Note from the single crossing property one can show there exists a disclosure policy $\bar{\delta}_h\left(\bar{\delta}^p\right) < \bar{\delta}_l\left(\bar{\delta}^p\right)$ such that $U^{P^h}\left(\lambda_h, \lambda_h, \bar{\delta}_h\left(\bar{\delta}^p\right)\right) > U^{P^h}\left(\lambda_h, q, \bar{\delta}^p\right)$ where P^h strictly prefers to deviate from the pooling equilibrium by signaling with $\bar{\delta}^p$ provided his type is correctly perceived. Since $\bar{\delta}_h\left(\bar{\delta}^p\right) < \bar{\delta}_l\left(\bar{\delta}^p\right)$ it follows $U^{P^l}\left(\lambda_l, \lambda_h, \bar{\delta}_h\left(\bar{\delta}^p\right)\right) < U^{P^l}\left(\lambda_l, \lambda_h, \bar{\delta}_l\left(\bar{\delta}^p\right)\right) = U^{P^l}\left(\lambda_l, q, \bar{\delta}_l\left(\bar{\delta}^p\right)\right)$. This implies P^l would not wish to mimic P^h and therefore the equilibrium, $\left\{\bar{\delta}_h = \bar{\delta}^p, \bar{\delta}_l = \bar{\delta}^p\right\}$, is not supported by the intuitive criteria. This completes our construction of the unque separating equilibrium for the $\rho > 0$ case. A similar argue may be used to construct the unque separating equilibrium corresponding to the $\rho < 0$ case as well. Summarizing we have:

Proposition 3 *For any equilibrium,*

> (i) *Suppose $\rho > 0$. There is a unique pareto dominate signaling equilibrium satisfying the Intuitive Criterion where P^h selects $\bar{\delta}^h < 1$ and P^l selects $\bar{\delta}^l = 1$.*

(ii) *Suppose* $\rho < 0$. *There is a unique pareto dominate signaling equilibrium satisfying the Intuitive Criterion where* P^l *selects* $\bar{\delta}^l < 1$ *and* P^h *selects* $\bar{\delta}^h = 1$.

4 Extension and implication

4.1 Heterogenous agents

We now relax one of the assumptions in the basic setup and assume that the agents vary in their private precision (relative to the public information precision). Specifically, we index agent i' private precision with β_i, and $\beta_i \leq \beta_j$ for $\forall i \leq j$. Further, define $\lambda_i \equiv \frac{\alpha}{\beta_i}$. As in the case of homogenous agents, agents do not know neither α nor β_i. Each agent i however understands that he face a planner with two possible levels of $\lambda_i \in \{\lambda_i^h, \lambda_i^l\}$, with $\lambda_i^h > \lambda_i^l > 1$, $\forall i$. The prior probability is that $\Pr(\lambda_i^h) = p$ and $\Pr\left(\lambda_i^l\right) = 1 - p$. The principal privately observes α and the agents' relative location i is public information. The following lemma characterizes agents' unique equilibrium effort supply in the case of heterogeneous agents.

Lemma 4: *A unique effort supply equilibrium exists:*

$$e_i = \begin{cases} s_i & \text{for} \quad i \notin D \\ e_i = w_i z + (1 - w_i)\, s_i & \text{for} \quad i \in D, \end{cases}$$

where

$$w_i = 1 - \frac{1 - \rho\bar{\delta}}{1 - \rho L} L_i$$

$$\text{with } L_i \equiv \frac{1}{\lambda_i + 1}, \; L \equiv \int_D L_i.$$

When agents differ in their private precision, their weight on public signal is not only a function of the measure of the inner circle ($\bar{\delta}$) but also the identity of the inner circle, as captured by the L term. Specifically, w_i decreases in L. That is, everything else equal, if the inner circle consists of agents with more precise private information, all agents weight the public signal less.

It can be readily verified that similar to the homogeneous agent case: 1) when λ_i is common knowledge, social welfare is maximized at a full disclosure policy with $\bar{\delta} = 1$

provided that ρ is not too negative, and 2) when $\lambda_i' s$ can only be privately observed by CP, incentive compatability issues arise: that is, when $\rho > 0$, CP wishes to pursuade agents that $\lambda = \lambda_i^h$; when $\rho < 0$ the social planner wishes to pursuade agents that $\lambda = \lambda_i^l$.[1] The new and remaining question is thus what is the "optimal" choice of inner circle agents such that a credible separation among different types of CP could be achieved. The following Proposition establishes this result.

Proposition 4 *There is a best separating equilibrium with these properties: (1) for $\rho > 0$ the optimal separating arrangement is characterized by $D_l^* = [0, 1]$ and the unique $D_h^* = [0, \bar{\delta}] \subset [0, 1]$ such that $PIC(L)$ is binding. (2) For $\rho < 0$ the optimal separating arrangement is characterized by $D_h^* = [0, 1]$ and the unique $D_l^* = [1 - \bar{\delta}, 1] \subset [0, 1]$ such that $PIC(H)$ is binding.*

Similar to the results in the previous section, the best separating arrangement in the case of heterogeneous agents involves information rationing. That is, high quality information is only disclosed to a selected inner circle. Additionally, Proposition 4 sheds light on characteristics of the inner circle: when agents' actions are strategic complements and agents under-coordinate their effort to the principal's desire, though a high precision CP is free to select her inner circle among any agents on $[0, 1]$, the best separation is achieved by only confiding with those who have the least precise private information. Intuitively, as before, information rationing leads to separation because the high precision CP exploits her superior ability to avoid large deviations of actions by the inner circle from those by the outer circle. And more important for the case of heterogeneous agents, such superior ability by the high precision CP is more pronounced when she discloses her signal z only to those least privately informed agents.

To elaborate, information rationing has two effects. First, believing the CP who rations information is of a high precision type, inner circle agents assign a larger weight on the disclosed signal z and a smaller weight on their own private signals than when the CP is perceived as having low precision. In other words, by credibly convey the CP's type, inner circle agents follow the signal z. Second, in comparison, since those outer circle agents do not observe z, they have to rely exclusively on their own private signals to make action choices. The key driving force here is that a high (low) precision CP expects her signal

[1] Contact the authors for details.

z to be more (less) correlated with the outer circle agent's signals. Thus, the high (low) precision CP expects a smaller (larger) deviation between actions taken by inner circle agents and those by outer circle agents. It is precisely this difference that prevents the low precision principal from mimicking.

Disclosing z to the least privately informed agents strengthens the two effects of information rationing. On the one hand, these agents' own private information is not very informative about the state and thus have to rely more on the CP's disclosed signal z to make action choices. In other words, a inner circle that consists of agents $\left[0, \bar{\delta}\right]$ follow the CP's disclosure z to a greater extent than if the inner circle consists of better informed agents. On the other hand, when those better informed agents whose private signals are quite informative about the state belong to the outer circle, their actions are expected to be correlated with the high precision CP's signal z to a greater extent than if the outer circle consists of the least informed agents whose signals contain a lot of noise. Thus, the net effect of disclosing z to agents $i \in \left[0, \bar{\delta}\right]$ is the enhanced ability of the high precision CP to escape large deviations between actions taken by the inner circle versus the outer circle, leading to a less costly separation arrangement.

In contrast, when agents' actions are strategic substitute and thus agents over-coordinate their efforts relative to the principal's desire, the low type CP discloses to a subset of inner circle to separate from the high type; and the optimal inner circle in this case is those agents with the most precise information.

Intuitively, again, although rationing information is welfare decreasing for both types of CPs, it is the high precision CP who suffers more. To elaborate, pick a single agent. If he doesn't receive the principal's signal, the expected loss for the principal due to the "Better Action" term is simply $\frac{1}{\beta_i}$ because this agent only bases his action choice on his private signal s_i.(We focus on the "Better Action" term because when $\rho < 0$ CP wishes to induce the agents to attach more importance to matching their actions with the state.) For a high type CP who mimics a low type one, her expected loss from the "Better Action" term by informing such an agent is

$$\frac{[\frac{1-\rho\bar{\delta}}{1-\rho L} L_i]^2}{\beta_i} + \frac{\left[1 - \frac{1-\rho\bar{\delta}}{1-\rho L} L_i\right]^2}{\alpha_h}.$$

Thus, the incremental loss for a mimicing high type CP from not disclosing the signal z

to this agent is

$$\frac{1}{\beta_i}-\frac{[\frac{1-\rho\bar{\delta}}{1-\rho L}L_i]^2}{\beta_i}-\frac{\left[1-\frac{1-\rho\bar{\delta}}{1-\rho L}L_i\right]^2}{\alpha_h}.$$

Similarly, the incremental loss for a low type type principal from not disclosing the signal z to this agent is

$$\frac{1}{\beta_i}-\frac{[\frac{1-\rho\bar{\delta}}{1-\rho L}L_i]^2}{\beta_i}-\frac{\left[1-\frac{1-\rho\bar{\delta}}{1-\rho L}L_i\right]^2}{\alpha_l}.$$

Easy to see, the mimicing high type incurs a higher incremental loss

$$\begin{aligned}&\frac{1}{\beta_i}-\frac{[\frac{1-\rho\bar{\delta}}{1-\rho L}L_i]^2}{\beta_i}-\frac{\left[1-\frac{1-\rho\bar{\delta}}{1-\rho L}L_i\right]^2}{\alpha_h}-(\frac{1}{\beta_i}-\frac{[\frac{1-\rho\bar{\delta}}{1-\rho L}L_i]^2}{\beta_i}-\frac{\left[1-\frac{1-\rho\bar{\delta}}{1-\rho L}L_i\right]^2}{\alpha_l})\\ =&\left[1-\frac{1-\rho\bar{\delta}}{1-\rho L}L_i\right]^2(\frac{1}{\alpha_l}-\frac{1}{\alpha_h})>0.\end{aligned}$$

Thus, the differential cost is higher for the mimicing high type when she tries to ration information. This again shows that rationing information is a credible separation device as in the homogeneous agents case. In addition, the above expression sheds light on which inner circle agents a low type CP wishes to choose in order to minimize the the cost of signalling. Note that

$$\frac{\partial\left[1-\frac{1-\rho\bar{\delta}}{1-\rho L}L_i\right]^2(\frac{1}{\alpha_l}-\frac{1}{\alpha_h})}{\partial\lambda_i}>0,\text{ when }\rho\text{ not too negative.}$$

This implies that the differential cost is increasing in λ.That is, withholding information from a low precision agent makes mimicing more costly for the high type CP.

4.2 Benefit of forced disclosure

The inefficient information dissemination implied by the unverifiability of the central planner's type may provide a justification for a commitment to requiring that all information be disclosed to all agents.

Let $U^P_{Pooling}$ denote the central planner's ex ante expected utility in a pooling situation when she doesn't know her type yet.

$$U^P_{Pooling}=qEU^p\left(\lambda_h,1,q\right)+(1-q)EU^p\left(\lambda_l,1,q\right)$$

where $EU^p\left(\lambda_j,1,q\right)$ is type λ_j $(j\in\{h,l\})$ central planner's expected utility in the pooling equilibrium when she discloses to everyone ($d = 1$), and when the agents believe that she is type λ_h (λ_l) with probability q ($1 - q$).

When the central planner's type is unknown, an inner circle agent optimally chooses action equal to the weighted average of his optimal action, i.e.

$$\begin{aligned} e_i^I &= q[\left(1 - w_i^I(\lambda_h)\right) s_i + w_i^I(\lambda_h) z] \\ &+(1-q)[\left(1 - w_i^I(\lambda_l)\right) s_i + w_i^I(\lambda_l) z] \\ &= (1 - \bar{m}) s_i + \bar{m} z, \qquad \text{where } \bar{m} \equiv q w_i^I(\lambda_h) + (1-q) w_i^I(\lambda_l). \end{aligned}$$

Given e_i^I and $e_i = s_i$,

$$EU^p(\lambda, 1, q) = \frac{1}{\beta}\left[\frac{w^2(2\rho - 1)}{\lambda} - (1 - w)^2\right].$$

Substituting the above into $U^P_{Pooling}$ yields

$$\begin{aligned} U^P_{Pooling} &= \frac{q}{\beta}[\frac{\bar{m}^2(2\rho-1)}{\lambda_h} - (1-\bar{m})^2] + \\ &\quad \frac{1-q}{\beta}[\frac{\bar{m}^2(2\rho-1)}{\lambda_l} - (1-\bar{m})^2] \end{aligned} \tag{8}$$

$$= \frac{q}{\beta}\frac{\bar{m}^2(2\rho-1)}{\lambda_h} + \frac{1-q}{\beta}\frac{\bar{m}^2(2\rho-1)}{\lambda_l} - \frac{(1-\bar{m})^2}{\beta} \tag{9}$$

Similarly, let $EU^p(\alpha, d, \alpha)$ to denote the central planner's expected utility in the best separating equilibrium. Since $D \leq 1$ in the separating equilibrium with strict inequality for any least one type, we have

$$EU^p(\lambda, D < 1, \lambda) \leq EU^p(\lambda, 1, \lambda) \equiv \frac{1}{\beta}\left[\frac{w^2(2\rho-1)}{\lambda} - (1-w)^2\right]$$

Denote $U^P_{Separating}$ as the central planner's ex ante expected utility in a separating situation when she doesn't know her type yet.

$$U^P_{Separating} \leq qEU^p(\lambda_h, 1, \lambda_h) + (1-q)EU^p(\lambda_l, 1, \lambda_l)$$

$$\begin{aligned} &= \frac{q}{\beta}\left[\frac{w_h^2(2\rho-1)}{\lambda_h} - (1-w_h)^2\right] + \\ &\quad \frac{1-q}{\beta}[\frac{w_l^2(2\rho-1)}{\lambda_l} - (1-w_l)^2] \end{aligned} \tag{10}$$

Subtract (10) from (8), we have

$$\begin{aligned} U^P_{pooling} - U^P_{Separating} \geq & \frac{(1-2\rho)}{\beta}\left[\frac{q}{\lambda_h}\left(w_h^2 - \bar{m}^2\right) + \frac{1-q}{\lambda_l}\left(w_l^2 - \bar{m}^2\right)\right] \\ & + \underbrace{[q(1-w_h)^2 + (1-q)(1-w_l)^2 - (1-\bar{m})^2]}_{>0 \text{ by concavity}}\frac{1}{\beta} \end{aligned} \quad (11)$$

When λ_l is large enough the first bracket goes to zero, hence $U^P_{pooling} - U^P_{Separating} > 0$ and

Proposition 5 *When λ_l is high enough, it is optimal to commit to a full disclosure policy for all $\rho < 1/2$.*

4.3 Incentive for ex-ante investment to acquire information

Suppose now that before knowing her own type, the planner could incur a cost of $c(q)$ to receive high quality information with probability q, where $c' > 0$, $c'' > 0$, $\lim_{q\to 1} c' = \infty$, and $\lim_{q\to 0} c' = 0$.

Proposition 6 *When $\rho > 0$, the central planner under-invests to acquire information. When $\rho < 0$, the central planner over-invests to acquire information.*

We start with when $\lambda's$ are publicly known (hence $d = 1$). Let $E\left[U^p|\lambda, d, \widehat{\lambda}\right]$ be the central planner's expected utility when her true type is α, when the disclosure set is D, and her perceived type is $\widehat{\alpha}$. The planner chooses q to maximize the expected value of the objective function:

$$\max_q qE\left(U^p|\lambda_h, 1, \lambda_h\right) + (1-q)\,E\left(U^p|\lambda_l, 1, \lambda_l\right) - c\left(q\right)$$

The first best level of q^{FB} is given by

$$c'\left(q^{FB}\right) = E\left(U^p|\lambda_h, 1, \lambda_h\right) - E\left(U^p|\lambda_l, 1, \lambda_l\right).$$

When $\lambda's$ are unknown, the central planner's expected utility depends on which type rations information in equilibrium. With strategic complementarity (i.e., $\rho > 0$), it's the high type that rations the information. Thus, the central planner chooses q to maximize:

$$\max_q qE\left(U^p|\lambda_h, D < 1, \lambda_h\right) + (1-q)\,E\left(U^p|\lambda_l, 1, \lambda_l\right) - c\left(q\right).$$

The optimal $q^{\rho>0}$ is given by

$$c'\left(q^{\rho>0}\right) = E\left(U^p|\lambda_h, D<1, \lambda_h\right) - E\left(U^p|\lambda_l, 1, \lambda_l\right).$$

Because, $E\left(U^p|\lambda_h, 1, \lambda_h\right) > E\left(U^p|\lambda_h, D<1, \lambda_h\right)$, it is easy to see that $q^{\rho>0} < q^{FB}$, that is, the central planner under-invests in acquiring information. This is because in order to separate from the low type, the high type central planner will end up restricting some agents access to her information, reducing the value of acquiring the information to begin with. As a result, the planner invests less than the first best level.

When the agents over-coordinate (i.e., $\rho < 0$), it is the low type that rations information. The central planner's optimal ex-ante investment in q is given by:

$$c'\left(q^{\rho<0}\right) = E\left(U^p|\lambda_h, 1, \lambda_h\right) - E\left(U^p|\lambda_l, D<1, \lambda_l\right).$$

Since $E\left(U^p|\lambda_l, 1, \lambda_l\right) > E\left(U^p|\lambda_l, 1, \lambda_l\right)$, we have $q^{\rho<0} > q^{FB}$, i.e., the central planner over-invests in acquiring information.

On a casual glimpse, this result seems counter-intuitive in that while ex post after the type is set, the high type central planner would like the agents to believe that she is of the lower type, ex ante the central planner actually over-invests to become the high type. This is because it is the low type that is forced to partially disclose despite the fact that full disclosure is the first best. Compared to the earlier case where the incentive to under-invest is due to the dampened reward of becoming a high type, the incentive to over-invest in this case is due to the hightened punishment for ending up as a low type.

4.4 Incentive for revealing ρ

The inability to directly communicate the quality of public information may also affect the central planner's incentive to reveal other relevant information such as the extent of the externality (ρ). Suppose the agents do not know ρ and know ρ can take only one of the two values ρ_l or ρ_h. Further assume these two types are of the same sign and are not too different from each other. Specifically, assume ratio of their absolute values is less than 2. To see the intuition better, we start with the case where λ is known and the central planner always disclose to all agents. The intuition is qualitatively similar when λ is unknown.

In general the central planner's incentive to reveal the true ρ is given by the sign of $\frac{\partial E\left(U^P|\rho,\widehat{\rho}\right)}{\partial \widehat{\rho}}$. When it is positive, low ρ central planner has incentive to lie to be the high ρ

type; when it is negative, high ρ central planner has incentive to pretend to be the low ρ type. Because the central planner's expected utility is quadratic in the agent's weight on z and maximizes at the socially optimal weight, the closer the agents' equilibrium weight is to the socially optimal level, the better off the central planner. This implies that

$$\begin{aligned}\frac{\partial E\left(U^P|\rho,\widehat{\rho}\right)}{\partial\widehat{\rho}} &\propto -\frac{\partial\left(\frac{\lambda}{\lambda+1-2\rho}-\frac{\lambda}{\lambda+1-\hat{\rho}}\right)^2}{\partial\widehat{\rho}} \\ &= \left(\frac{\lambda}{\lambda+1-2\rho}-\frac{\lambda}{\lambda+1-\hat{\rho}}\right)\frac{\partial E\left(\frac{\lambda}{\lambda+1-\hat{\rho}}\right)}{\partial\widehat{\rho}} \\ &\propto (2\rho-\widehat{\rho})\,|_{\rho=\hat{\rho}}=\rho \qquad (12)\end{aligned}$$

Thus, when $\rho > 0$, (12) is positive, suggesting that on the margin, the low ρ_l planner would benefit if the agents choose their action believing their preference is given by ρ_h (assuming $2\rho_l - \rho_h > 0$). The ρ_h type also benefits if the agents believe she is of a type higher than ρ_h (but not too high). Similarly, when $\rho < 0$, assume $2\rho_h < \rho_l$, then (12) is negative, suggesting that on the margin, the high ρ_h central planner would benefit if the agents believe they have a low ρ_l. This leads to the next Proposition.

Proposition 7 *When $\rho > 0$, the ρ_l central planner has incentive to lie to be ρ_h provided $2\rho_l - \rho_h > 0$; and when $\rho < 0$ the ρ_h central planner has incentive to lie to be ρ_l provided $2\rho_h - \rho_l < 0$.*

The intuition is the following. With strategic complementarity, the agents under-coordinate relative to the first best level, the central planner has incentive to induce the agents to put more weight on the public signal. Since the agents' weight on the public signal is increasing in ρ, the ρ_l central planner would have incentive to lie to be ρ_h. Similarly, when $\rho < 0$, the agents over-coordinate and over-weight the public signal. This gives the ρ_h planner incentive to lie to be ρ_l.

5 Conclusion

To be completed.

6 PROOFS

Proof of Propostion 2:

Express the social welfare function as

$$\frac{U\left(\bar{\delta}\right)}{2}=\frac{1}{2}\left[\frac{\bar{\delta}-1}{\beta}-\frac{\bar{\delta}w^2}{\alpha}-\frac{\bar{\delta}\left(1-w\right)^2}{\beta}\right]+\frac{\rho\bar{\delta}^2w^2}{\alpha}.$$

Further $\frac{\partial w}{\partial \delta}=\frac{\rho\hat{\lambda}}{\left(1+\hat{\lambda}-\rho\delta\right)^2}=\frac{\rho w}{\left(1+\hat{\lambda}-\rho\delta\right)}$. Further

$$\begin{aligned}
\frac{\partial U^P}{\partial \delta} &= \frac{1}{2}\left[\frac{1-\left(1-w\right)^2}{\beta}-\frac{w^2}{\alpha}\right]+\frac{2\rho\bar{\delta}w^2}{\alpha}+\frac{2\rho\bar{\delta}^2w}{\alpha}\frac{\partial w}{\partial \delta}+\left[\frac{\bar{\delta}\left(1-w\right)}{\beta}-\frac{\bar{\delta}w}{\alpha}\right]\frac{\partial w}{\partial \delta}\\
&= \frac{1-\left(1-w\right)^2}{2\beta}-\frac{w^2}{2\alpha}+\frac{2\rho\bar{\delta}w^2}{\alpha}\left(1+\frac{\bar{\delta}\rho}{\left(1+\hat{\lambda}-\rho\delta\right)}\right)+\frac{\bar{\delta}\rho w}{\left(1+\hat{\lambda}-\rho\delta\right)}\left[\frac{\left(1-w\right)}{\beta}-\frac{w}{\alpha}\right]\\
&= \frac{w}{\alpha}\left\{\frac{\lambda\left(2-w\right)-w}{2}+2\rho\bar{\delta}w\left(\frac{1+\hat{\lambda}}{\left(1+\hat{\lambda}-\rho\delta\right)}\right)+\frac{\bar{\delta}\rho}{\left(1+\hat{\lambda}-\rho\delta\right)}\left[\left(1-w\right)\lambda-w\right]\right\}\\
&= \frac{w}{\alpha}\left\{\frac{\lambda\left(2-w\right)-w}{2}+\frac{\bar{\delta}\rho}{\left(1+\hat{\lambda}-\rho\delta\right)}\left[2w\left(1+\hat{\lambda}\right)+\left(1-w\right)\lambda-w\right]\right\}
\end{aligned}$$

Easy to verify that $2w\left(1+\hat{\lambda}\right)+\left(1-w\right)\lambda-w>0$, so the above is positive for sure when $\rho>0$.

When $\rho<0$, the first term is positive while the second term is monotonically increasing in ρ but equals to zero when $\rho=0$. Thus as long as ρ is not too negative, $\frac{\partial U^P}{\partial \delta}$. More precise bound can be shown by further simplifying the terms in the braces above as

$$\begin{aligned}
* &=_s \frac{\left(\lambda\left(2-w\right)-w\right)}{2}\left(1+\hat{\lambda}-\rho\delta\right)+\bar{\delta}\rho\left[2w\left(1+\hat{\lambda}\right)+\left(1-w\right)\lambda-w\right]\\
&= \frac{\left(\lambda\left(2-w\right)-w\right)}{2}\left(1+\hat{\lambda}\right)+\bar{\delta}\rho\left[2w\left(1+\hat{\lambda}\right)+\left(1-w\right)\lambda-w-\frac{\left(\lambda\left(2-w\right)-w\right)}{2}\right]\\
&=_s \frac{\left(\lambda\left(2-w\right)-w\right)}{2}+\bar{\delta}\rho w\left[2-\frac{1+\lambda}{2\left(1+\hat{\lambda}\right)}\right]\\
&> \frac{\left(\lambda\left(2-w\right)-w\right)}{2}+2\rho
\end{aligned}$$

where the last inequality follows because $w\left[2-\frac{1+\lambda}{2\left(1+\hat{\lambda}\right)}\right]$ is bounded above by 2. Thus, as long as

$$\rho > -\frac{1}{2}\frac{\left(\lambda\left(2-w\right)-w\right)}{2}$$

we are set. A sufficient condition is

$$\rho > -\frac{2\lambda-\left(1+\lambda\right)}{4} = -\frac{\lambda-1}{4}.$$

For $\rho > -\frac{1}{2}$ to be a sufficient condition, we need $\frac{(\lambda(2-w)-w)}{2} > 1$, a sufficient condition for which is $\lambda > \frac{7}{5}$. ■

When λ is known, replace $\hat{\lambda}$ with λ before differentiating EU^P with respect to $\bar{\delta}$, we obtain

$$\frac{dEU^P}{d\bar{\delta}} =_s w^2 + 2w\bar{\delta}\frac{dw}{d\bar{\delta}} =_s 1 + \frac{2\rho\bar{\delta}}{\lambda+1-\rho\bar{\delta}} =_s 1+\lambda+\rho\bar{\delta} > 0$$

for all $\rho > -1-\lambda$.

PROOF OF LEMMA 1

Follows previous proofs and probably may be omitted

PROOF OF LEMMA 2

Social surplus as a function of the Disclosure policy $\bar{\delta}$ is given by

$$U_i^P\left(v\right) = \frac{v^2}{2} - \frac{L_i\left(\bar{\delta}_i\right)}{2}$$

$$\text{where } L_j\left(\bar{\delta}_j\right) = \frac{\left(\bar{\delta}_j\left(1-2\rho\bar{\delta}_j\right)w^2\left(q\left(\bar{\delta}_j\right)\right)+\bar{\delta}_j\lambda_j\left(\left(1-w\left(q\left(\bar{\delta}_j\right)\right)\right)^2+\left(1-\bar{\delta}_j\right)\right)\right)}{\alpha_j}$$

$$w\left(q\right) = q\frac{\lambda_h}{\lambda_h+\left(1-\rho\right)\bar{\delta}_j} + \left(1-q\right)\frac{\lambda_l}{\lambda_l+\left(1-\rho\right)\bar{\delta}_j}$$

Differentiating $L_i\left(\bar{\delta}_i\right)$ with respect to q we obtain

$$\begin{aligned} -\frac{L_i\left(\bar{\delta}_i\right)}{dq} &= -\frac{1}{\alpha_j}\left(\bar{\delta}_j 2w\left(1-2\rho\bar{\delta}_j\right)-2\bar{\delta}_j\lambda_j\left(1-w\right)\right)\frac{dw\left(q\right)}{dq} \\ &= \lambda_j\left(1-w\right)-w\left(1-2\rho\bar{\delta}_j\right)\left(\frac{dw\left(q\right)}{dq}\frac{2\bar{\delta}_j}{\alpha_j}\right) \end{aligned}$$

Also note that

$$\begin{aligned}\frac{\lambda_h}{\lambda_h+(1-\rho)\bar{\bar{\delta}}} &\geq w \geq \frac{\lambda_l}{\lambda_l+(1-\rho)\bar{\bar{\delta}}} \\ \frac{(1-\rho)}{\lambda_l+(1-\rho)\bar{\bar{\delta}}} &\geq 1-w \geq \frac{(1-\rho)\bar{\delta}}{\lambda_h+(1-\rho)\bar{\bar{\delta}}}\end{aligned}$$

Then for $\rho > 0$, we have

$$\begin{aligned}-\frac{L_i\left(\bar{\delta}_i\right)}{dq} &> \frac{\lambda_j(1-\rho)\bar{\delta}}{\lambda_h+(1-\rho)\bar{\bar{\delta}}}-\frac{\lambda_h\left(1-2\rho\bar{\delta}_j\right)}{\lambda_h+(1-\rho)\bar{\bar{\delta}}} \\ &\Rightarrow -\frac{L_i\left(\bar{\delta}_i\right)}{dq}>0 \text{ for } \lambda_j=\lambda_h \\ &\Rightarrow -\frac{L_i\left(\bar{\delta}_i\right)}{dq}>0 \text{ for } \frac{\lambda_l}{\lambda_h} \geq \frac{\left(1-2\rho\bar{\delta}\right)}{(1-\rho)\bar{\delta}}\end{aligned}$$

For $\rho < 0$ we can establish by a similar argument the following

$$\begin{aligned}-\frac{L_i\left(\bar{\delta}_i\right)}{dq} &< \frac{\lambda_j(1-\rho)\bar{\delta}}{\lambda_l+(1-\rho)\bar{\bar{\delta}}}-\frac{\lambda_l\left(1-2\rho\bar{\delta}_j\right)}{\lambda_l+(1-\rho)\bar{\bar{\delta}}} \\ &\Rightarrow -\frac{L_i\left(\bar{\delta}_i\right)}{dq}<0 \text{ for } \lambda_j=\lambda_l \\ &\Rightarrow -\frac{L_i\left(\bar{\delta}_i\right)}{dq}<0 \text{ for } \frac{\lambda_l}{\lambda_h} \leq \frac{(1-\rho)\bar{\delta}}{\left(1-2\rho\bar{\delta}\right)}\end{aligned}$$

■

Proof for Proposition 4:

Part I. Partial disclosure as separating equilibrium

We first establish the case where $\rho > 0$. Let's start with a relaxed program where we ignore PIC(H). Later, we'll show that the optimal solution under the relaxed program satisfies PIC(H). The proof proceeds in several steps.

- (Partial Disclosure Hurts) implies that $E[U^p|\lambda_l, D_l, \lambda_l]$ is strictly increasing in D_l, i.e., when a type λ_l central planner truthfully chooses D_l, the low type central planner's payoff strictly increases with D_l. Similarly, $E[U^p|\lambda_h, D_h, \lambda_h]$ and $E[U^p|\lambda_l, D_h, \lambda_h]$ are also strictly increasing in D_h.

- The optimal separating arrangement has $D_l^* = 1$. Suppose otherwise. Increasing D_l slightly increases the RHS of $PIC(L)$, makes it slack and improves upon the low type central planner's payoff.

- (Incentives to Lie) shows that the optimal separating arrangement cannot have $D_h^* = 1$. Otherwise, the low precision central planner would have incentives to pretend to be a high type.

- PIC(L) is binding at the optimal. Suppose otherwise. Slightly increase D_h while keeping PIC(L) still slack increases the high type central planner's payoff. A contradiction.

  - The unique optimal $D_h^* > 0$. Easy to establish that $E[U^p|\lambda_l, 0, \lambda_h] < E[U^p|\lambda_l, 1, \lambda_l]$ (See below). In addition, from (Incentives to Lie), $E[U^p|\lambda_l, 1, \lambda_h] > E[U^p|\lambda_l, 1, \lambda_l]$; and from (Partial Disclosure Hurts), $E[U^p|\lambda_l, D_h, \lambda_h]$ is strictly increasing in D_h. Thus there exists $D_h \in (0, 1)$ such that $PIC(L)$ is binding.

  - Let's now verify that $PIC(H)$ is satisfied by the solution to relaxed program. Suppose otherwise. There are three possibilities.

1. First possibility: PIC(H) is the only binding constraint at the optimal solution. That is,

$$\begin{aligned} E[U^p|\lambda_h, D_h, \lambda_h] &= E[U^p|\lambda_h, D_l, \lambda_l] \\ &\quad \textit{and} \\ E[U^p|\lambda_l, D_l, \lambda_l] &> E[U^p|\lambda_l, D_h, \lambda_h]. \end{aligned}$$

   From the first and fifth bullet point, $\forall D_l \subset [0, 1]$, $E[U^p|\lambda_l, D_l, \lambda_l] < E[U^p|\lambda_l, [0, 1], \lambda_l] < E[U^p|\lambda_l, [0, 1], \lambda_h]$. This implies, to satisfy $E[U^p|\lambda_l, D_l, \lambda_l] > E[U^p|\lambda_l, D_h, \lambda_h]$, $D_h \neq [0, 1]$. Thus, slightly increasing D_h increases the high type planner's payoff while satisfying both constraint. A contradiction.

2. Second possibility: both PIC(H) and PIC(L) are binding at the optimal solution. That is,

$$E[U^p|\lambda_h, D_h, \lambda_h] = E[U^p|\lambda_h, D_l, \lambda_l] \tag{13}$$

$$E[U^p|\lambda_l, D_l, \lambda_l] = E[U^p|\lambda_l, D_h, \lambda_h]. \tag{14}$$

   When the solution of the relaxed program $\{D_l^*, D_h^*\}$ satisfies both (13) and (14), then it is without of loss of generality to ignore PIC(H). When the solution of the relaxed program doesn't satisfy (13), then the solution to (13) and (14) $\{D_l^{**}, D_h^{**}\}$ must have $D_l^* = [0,1] \supset D_l^{**}$. Hence, slightly increasing D_l^{**} increases the low precision central planner's payoff while satisfying both constraint. A contradiction.

3. Third, at the optimal solution, $D_l \neq [0,1]$ and

$$E[U^p|\lambda_h, D_h, \lambda_h] > E[U^p|\lambda_h, D_l, \lambda_l]$$

$$E[U^p|\lambda_l, D_l, \lambda_l] = E[U^p|\lambda_l, D_h, \lambda_h].$$

Clearly, slightly increasing D_l increases the low precision central planner's payoff while satisfying both constraint. A contradiction.

- The optimal $D_h^* = [0, d^*]$. See Part II for detailed proof.

Some details:

From (??),

$$\begin{aligned} E[U^p|\lambda_l, 0, \lambda_h] &= -\int_{[0,1]} \frac{1}{\beta_i} \\ E[U^p|\lambda_l, 1, \lambda_l] &= 2\rho \frac{\left(\int_{[0,1]} w_i di\right)^2}{\alpha_l} - \int_{[0,1]} \left(\frac{w_i^2}{\alpha_l} + \frac{(1-w_i)^2}{\beta_i} \right) \end{aligned}$$

Hence

$$\begin{aligned} & E[U^p|\lambda_l, 0, \lambda_h] - E[U^p|\lambda_l, 1, \lambda_l] \\ = & \int_{[0,1]} \left(\frac{w_i^2}{\alpha_l} + \frac{(1-w_i)^2 - 1}{\beta_i} \right) di - 2\rho \frac{\left(\int_{[0,1]} w_i di\right)^2}{\alpha_l} \\ < & \int_{[0,1]} \left(\frac{w_i^2}{\beta_i} + \frac{(1-w_i)^2 - 1}{\beta_i} \right) di - 2\rho \frac{\left(\int_{[0,1]} w_i di\right)^2}{\alpha_l} \\ = & \int_{[0,1]} \left(\underbrace{\frac{2w_i(w_i - 1)}{\beta_i}}_{<0} \right) di - 2\rho \frac{\left(\int_{[0,1]} w_i di\right)^2}{\alpha_l} \end{aligned}$$

Clearly, the last expression is negative when $\rho > 0$.

When $\rho < 0$, we can similarly get

$$\begin{aligned}
& E[U^p|\lambda_h, 0, \lambda_l] - E[U^p|\lambda_h, 1, \lambda_h] \\
= & \int_{[0,1]} \left(\frac{w_i^2}{\alpha_h} + \frac{(1-w_i)^2 - 1}{\beta_i} \right) di - 2\rho \frac{\left(\int_{[0,1]} w_i di\right)^2}{\alpha_h} \\
< & \int_{[0,1]} \left(\frac{w_i^2}{\beta_i} + \frac{(1-w_i)^2 - 1}{\beta_i} \right) di - 2\rho \frac{\left(\int_{[0,1]} w_i di\right)^2}{\alpha h} \\
= & \int_{[0,1]} \left(\frac{2w_i(w_i - 1)}{\beta_i} \right) di - 2\rho \frac{\left(\int_{[0,1]} w_i di\right)^2}{\alpha_h}
\end{aligned}$$

Because $\frac{2w_i(w_i-1)}{\beta_i} < 0$ and is bounded away from 0, the last expression is negative when ρ is not too negative.

Part II: Whom to Disclose

To ease exposition, define the following terms:

$$\begin{aligned}
W &= C - D \\
C &= w_x \frac{2-\rho L}{1-\rho L} \rho \left(\int_D w_i \right) \\
D &= \frac{w_x^2}{2} \\
N &= \frac{1-(1-w_x)^2}{2x} \\
M &= w_x \frac{\rho k}{(1-\rho L)} \left(\frac{\alpha - \hat{\alpha}}{\hat{\alpha}} \right) \int_D P_i L_i di \\
q &= \frac{x}{y} \frac{(2-w_y)}{(2-w_x)} < 1 \\
h &= \frac{C - \left(\frac{w_y}{w_x}\right) D}{C - D} \\
\Delta &= \frac{w_x}{2} \left(\frac{1}{x} - \frac{1}{y} \right)
\end{aligned}$$

Note:

- $|W|$ is bounded as

$$|W| < |C| < \frac{2-\rho L}{1-\rho L} |\rho| \delta < \frac{2-L}{1-L}$$

  the last inequality follows because $\frac{2-\rho L}{1-\rho L}\rho$ is monotonically increasing in ρ.

- $q < 1$ because
$$\frac{2-w_x}{x} = \frac{1+kL_x}{x} = \frac{1}{x} + k\frac{1}{x+\hat{\alpha}}$$
is decreasing in x.

- $M = 0$ when $\alpha = \hat{\alpha}$; and $M < 0$ otherwise. This is because $sign\,(M) = sign\,(\rho\,(\alpha - \hat{\alpha}))$ and when $\rho < (>)\,0$, it's the high (low) type that has incentive to mimic low (high) type.

$$\begin{aligned}
-\frac{\partial U^P(\alpha,\hat{\alpha})}{\partial x} &= -\frac{\partial U^P(\alpha)}{\partial x} - \frac{\partial U^P(\alpha)}{\partial \delta}\frac{d\delta}{dx} \\
&= w_x\frac{2\rho}{\alpha}\left(\int_D w_i\right) + \frac{1}{2}\left[\frac{1}{x} - \left(\frac{w_x^2}{\alpha} + \frac{(1-w_x)^2}{x}\right)\right] \\
&\quad -\frac{2\rho}{\alpha}\left(\int_D w_i\right)\int_D \frac{\partial w_i}{\partial x}di + \int_D\left[\frac{w_i}{\alpha} - \frac{(1-w_i)}{\beta_i}\right]\frac{\partial w_i}{\partial x}di
\end{aligned}$$

Substitute in

$$\frac{\partial w_i}{\partial x} = -\frac{\partial\frac{(1-\rho\delta)L_i}{1-\rho L}}{\partial x} = -\frac{\rho L_i}{(1-\rho L)}\frac{(1-\rho L)+(1-\rho\delta)(-L_x)}{(1-\rho L)} = -\frac{\rho L_i}{(1-\rho L)}t_x \tag{15}$$

and (**??**), collect terms:

$$\begin{aligned}
-\frac{\partial U^P(\alpha,\hat{\alpha})}{\partial x} &= \frac{w_x}{\alpha}\frac{2-\rho L}{1-\rho L}\rho\left(\int_D w_i\right) - \frac{w_x^2}{2\alpha} + \frac{1-(1-w_x)^2}{2x} \\
&\quad + \frac{w_x}{\alpha}\frac{\rho k}{(1-\rho L)}\left(\frac{\alpha-\hat{\alpha}}{\hat{\alpha}}\right)\int_D P_i L_i di \\
&= \frac{1}{\alpha}(W + M + \alpha N)
\end{aligned}$$

- Similarly,
$$\frac{\partial U^P(\alpha,\hat{\alpha})}{\partial y} = \frac{1}{\alpha}\frac{w_y}{w_x}\left[hW + M + \alpha q N\right].$$

When $\rho < 0$:

Let's consider the case $\rho < 0$. Suppose not. There must exist a mass of informed agents with positive measure $[x, y]$ with $1 > y > x$, and a continuous mass of uninformed agents just immediately above y. Now, let's slightly increase x and y such that PIC(H) still binding. Mathematically, PIC(H) binding implicitly defines x as a function of y:

$$\frac{dx}{dy} = -\frac{\frac{\partial LHS(PIC(H))}{\partial y}}{\frac{\partial LHS(PIC(H))}{\partial x}} = \frac{\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_l)}{\partial y}}{-\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_l)}{\partial x}} > 0.$$

The net effect on the low type principal's payoff is

$$\Delta = \frac{\partial E\left(U|\alpha_l, D = [x, y], \widehat{\alpha} = \alpha_l\right)}{\partial y} + \underbrace{\frac{\partial E\left(U|\alpha_l, D = [x, y], \widehat{\alpha} = \alpha_l\right)}{\partial x}}_{<0}\frac{dx}{dy} >?0$$

$$\Leftrightarrow \frac{\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_l)}{\partial y}}{-\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_l)}{\partial x}} = \frac{dx}{dy} <? \frac{\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_l)}{\partial y}}{-\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_l)}{\partial x}}$$

Since the demoninators are both positive, the above is equivalent to the following expression being negative

$$\begin{aligned}
& \underbrace{\frac{1}{\alpha_h}\frac{w_y}{w_x}\left[hW + M + \alpha_h qN\right]}_{\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_l)}{\partial y}}\underbrace{\frac{1}{\alpha_l}\left[W + \alpha_l N\right]}_{-\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_l)}{\partial x}} - \underbrace{\frac{1}{\alpha_l}\frac{w_y}{w_x}\left[hW + \alpha_l qN\right]}_{\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_l)}{\partial y}}\underbrace{\frac{1}{\alpha_h}\left[W + M + \alpha_h N\right]}_{-\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_l)}{\partial x}} \\
= \; & {}^s hW^2 + a_l hNW + MW + a_l MN + a_h qNW + a_h a_l qN^2 - \\
& \left(hW^2 + hMW + a_h hNW + a_l qNW + a_l qMN + a_l a_h qN^2\right) \\
= \; & \left(a_l - a_h\right) hNW + (1 - h) MW + a_l MN (1 - q) + \left(a_h - a_l\right) qNW \\
= \; & \left(a_h - a_l\right) NW (q - h) + M\left[a_l N (1 - q) + (1 - h) W\right]
\end{aligned}$$

Simplify the second term (see details below), we have

$$a_l N (1 - q) + (1 - h) W = \alpha_l \Delta > 0$$

and therefore

$$W (q - h) = (q - 1)(W + a_l N) + \alpha_l \Delta$$

Substitute it in, we have

$$\begin{aligned}
& \left(a_h - a_l\right) N\left[(q - 1)(W + a_l N) + \alpha_l \Delta\right] + M a_l \Delta \\
= \; & {}^s (q - 1) N (W + a_l N) + \left[N + \frac{M}{(a_h - a_l)}\right] a_l \Delta <?0 \\
\Leftrightarrow \; & \left[N a_l + \frac{M a_l}{(a_h - a_l)}\right] \Delta <? (1 - q) N (W + a_l N)
\end{aligned}$$

Divide both sides by Δ, note that $\frac{(1-q)N}{\Delta} = 1 + kP_x P_y > 1$ (details below), and substitute in the expression for M, we have

$$kP_xP_yNa_l >? \frac{w_x}{2} \frac{\rho k}{(1-\rho L)} \int_D P_i L_i di - (1 + kP_xP_y) W$$

Since the RHS of the inequality is bounded while the LHS can go to ∞ when $\alpha_l \to \infty$, the above is true when α is large enough.

When $\rho > 0$:

Let's consider the case $\rho > 0$. Suppose not. There must exist a mass of informed agents with positive measure $[x, y]$ with $y > x > 0$, and a continuous mass of uninformed agents just immediately below x. Now, let's slightly decrease x and y such that PIC(L) still binding. Mathematically, PIC(L) binding implicitly defines x as a function of y:

$$\frac{dx}{dy} = -\frac{\frac{\partial LHS(PIC(L))}{\partial y}}{\frac{\partial LHS(PIC(L))}{\partial x}} = -\frac{\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_h)}{\partial y}}{\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_h)}{\partial x}} > 0.$$

The net effect of changing x on the high type principal's payoff is

$$\Delta = \frac{\partial E\left(U|\alpha_h, D = [x, y], \widehat{\alpha} = \alpha_h\right)}{\partial x} + \underbrace{\frac{\partial E\left(U|\alpha_h, D = [x, y], \widehat{\alpha} = \alpha_h\right)}{\partial y}}_{<0} \frac{dy}{dx} <?0$$

$$0 < \frac{-\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_h)}{\partial x}}{\frac{\partial E(U|\alpha_l, D, \widehat{\alpha}=\alpha_h)}{\partial y}} = \frac{dy}{dx} <? \frac{-\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_h)}{\partial x}}{\frac{\partial E(U|\alpha_h, D, \widehat{\alpha}=\alpha_h)}{\partial y}}$$

Substitute the expressions for these derivatives from above, we need to show the following expression is negative:

$$\begin{aligned}
&(W+M+\alpha_l N)(hW+a_h qN)-(hW+M+a_l qN)(W+a_h N)\\
&=hW^2+a_h qNW+hMW+a_h qMN+a_l hNW+a_l a_h qN^2\\
&-hW^2-a_h hNW-MW-a_h MN-a_l qNW-a_l a_h qNh2\\
&=a_h NW(q-h)+MW(h-1)+a_h MN(q-1)+a_l NW(h-q)\\
&=(a_h-a_l)NW(q-h)+M(a_h N(q-1)+W(h-1))\\
&=(a_h-a_l)NW(q-h)+Ma_h\Delta\\
&=^s (q-1)N(W+a_h N)+\left[N+\frac{M}{(a_h-a_l)}\right]a_h\Delta <?0\\
&\Leftrightarrow \left[Na_h+\frac{Ma_h}{(a_h-a_l)}\right]\Delta <?(1-q)N(W+a_h N)\\
&\Leftrightarrow kP_xP_yNa_h >?\frac{w_x}{2}\frac{\rho k}{(1-\rho L)}\int_D P_iL_idi-(1+kP_xP_y)W
\end{aligned}$$

The LHS can go to ∞ when $\alpha\to\infty$ while the RHS is bounded from above. Thus, as long as α_h is large enough, the above is true.

Details for simplifying $aN(1-q)+(1-h)W$:

$$\begin{aligned}
& aN(1-q)+(1-h)W\\
=\;& \alpha\left(\frac{w_x(2-w_x)}{2x}-\frac{w_x(2-w_x)}{2x}\frac{x}{y}\frac{(2-w_y)}{(2-w_x)}\right)-\frac{w_x^2}{2}\left(\frac{w_x-w_y}{w_x}\right)\\
=\;& \frac{w_x}{2}\left[\alpha\left(\frac{(2-w_x)}{x}-\frac{(2-w_y)}{y}\right)-(w_x-w_y)\right]\\
=\;& \frac{w_x}{2}\left[\alpha\left(\frac{1}{x}-\frac{1}{y}+k\left(\frac{1}{x+\hat{\alpha}}-\frac{1}{y+\hat{\alpha}}\right)\right)-k\left(\frac{y}{y+\hat{\alpha}}-\frac{x}{x+\hat{\alpha}}\right)\right]\\
=\;& \frac{w_x}{2}\left[\alpha\left(\frac{1}{x}-\frac{1}{y}+k\frac{y-x}{(x+\hat{\alpha})(y+\hat{\alpha})}\right)-\hat{\alpha}k\frac{y-x}{(x+\hat{\alpha})(y+\hat{\alpha})}\right]\\
=\;& \alpha\frac{w_x}{2}\left(\frac{1}{x}-\frac{1}{y}\right)+\frac{w_x}{2}(\alpha-\hat{\alpha})k\frac{y-x}{(x+\hat{\alpha})(y+\hat{\alpha})}|_{\alpha=\hat{\alpha}}\equiv\alpha\Delta>0
\end{aligned}$$

Details for simplifying $\frac{(1-q)N}{\Delta}$:

$$
\begin{aligned}
\frac{(1-q)\,N}{\Delta} &= \frac{\left(1-\frac{x}{y}\frac{2-w_y}{2-w_x}\right)\frac{w_x(2-w_x)}{2x}}{\frac{w_x}{2}\left(\frac{1}{x}-\frac{1}{y}\right)} = \frac{\frac{2-w_x}{x}-\frac{2-w_y}{y}}{\frac{1}{x}-\frac{1}{y}} \\
&= \frac{\frac{1+kL_x}{x}-\frac{1+kL_y}{y}}{\frac{1}{x}-\frac{1}{y}} = \frac{\frac{1}{x}+k\frac{1}{x+a}-\frac{1}{y}-k\frac{1}{y+a}}{\frac{1}{x}-\frac{1}{y}} \\
&= 1+\frac{k\frac{y-x}{(x+a)(y+a)}}{\frac{y-x}{xy}} = 1+kP_xP_y > 1
\end{aligned}
$$

■